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Hybrid Quark Model and Six-Quark Probability

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Abstract

In the hybrid quark model (HQM), when the distance between the two nucleons is greater than a certain cut off radius, they maintain their identity as nucleons but when the distances between the two baryons is smaller than a certain cut off radius, then the baryons overlap to form a six-quark bag. The fact that the probability to find six-quark for $r < r_0$ *does not have to be the same as that of finding two nucleons for* $r < r_0$ *can be accommodated either by allowing for a different normalization of the external wave function or by modifying the potential for* $r > r_0$. *The mathematical framework for the evaluation of six-quark probability for nucleon-nucleon pair inside a nucleus or a hypernucleus from the shell model wave functions for the evaluation of matrix elements in the overlap probability has been compared.*

Keywords: *Quantum chromodynamics(QCD), quark degrees of freedom, six-quark bag*

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INTRODUCTION

The correct theory which describes the quark degrees of freedom in nucleons explicitly is quantum chromodynamics (QCD) [1,2], but this is very difficult. Despite great progress in its perturbative [3]and non-perturbative [4,5] aspects there are only two basic pieces of information available, first the quarks have the property of asymptotic freedom, that is when they are close together they do not interact, second quarks are confined, which means when the separation distance between quarks is large the interaction between them is strong and attractive. This means quark, gluon and any object carrying color are confined. Thus if one nucleon carrying quark, gluon and pions is to interact with another nucleon that is well separated from it, all terms save the exchanges of color singlets are prohibited by the confinement property. Since the pion is the lightest color singlet object formed from a quark and antiquark pair, one expects the exchange of single pion to dominate the long range NN interaction. One also expects considerable contribution from the exchange of two pions. Thus at large separation distances the conventional picture of mesonic exchanges is in agreement with the QCD, but for small separation between nucleons the quarks in one nucleon overlap with the quarks in the other nucleon and this should be treated as a system of confined quarks interacting via various perturbative terms of QCD.

In order to bridge the gap between the short distance perturbative QCD region and the long range pion exchange force [6] developed a hybrid quark model which retains the conventional meson exchange picture at long distances and represents the effect of QCD at short distances. This model is based on the coordinate space representation of nuclear systems. There external regions in which separated baryons are represented as color singlets interaction through forces arising from the exchange of color singlet objects like pions. In the internal regions the quarks associated with two or more baryons interact with full color freedom.

Thus in hybrid quark model, the nuclear matter has two phases. The nucleons are assumed to maintain their identity and properties as long as the distance between them is greater than a certain critical radius r_0 and if distance between two nucleons is less than r_0 the system is treated as six-quarks.

FORMALISM

According to hybrid quark model (HQM) the two baryons overlap and form a six-quark bag when the distance between them is smaller than a certain cut off radius r_0 . Then the wavefunction for the six-quark system, with ξ as other internal variables can be representedin the following equations, given by (Greben and Thomas, 1984) [7].

$$\phi(r_1....r_6) = \phi_6(r,\xi) \qquad r < r_0 \qquad (1)$$

The six-quark probability can be defined as,

$$P_{6q} = |C|^2 \int \phi_6(r_1.....r_6)\, dr_1..........dr_6 \qquad (2)$$

$$= |C|^2 \int |\phi_6(r,\xi)|^2 d^3r d\xi \qquad (3)$$

Exact calculation of P_{6q} within the constraints of QCD in difficult to make in a model independent way, but P_{6q} can be related to the external NN wave functions under different approximations.The conservation of probability current across the boundary of matching radius at $r = r_0$ demands,

$$\frac{\partial \rho}{\partial t} = -\nabla . j \qquad 4)$$

$$r.j\big|_{r=r_0-\varepsilon} = r.j\big|_{r=r_0+\varepsilon} \qquad (5)$$

Integrating the current conservation equation [4] over ξ and r with $r < r_0$,

$$\int d\xi d^3r\theta(r_0 - r)\frac{\partial \rho}{dt} = -\int d\xi d^3r\theta(r_0 - r)\nabla . j \qquad (6)$$

Except for the time derivative, the left-hand side resembles P_{6q}. In the right-hand side the volume integral can be replaced by surface integral by the use of divergence theorem, so that the right-hand side of equation [6] depends on *r.j*. Now using the equation of continuity of current equation [5] the right-hand side of equation [6] can be expressed in terms of $\phi_{12}(r)$. Thus a very difficult expression involving six-quarks can be expressed in terms of ordinary nucleonic wavefunction evaluated at the boundary $r = r_0$, using the probability current conservation and the equation of continuity.

Since we are interested mostly with the state of the valence particle to form six-quark bag with the core particles only, the six-quark probability can be calculated from the shell model wave functions. Thus the probability of the valence particle being part of one of more six-quark bags with the core nucleons is,

$$P_Q^{6q} = \langle \Psi^0 | \Psi^0 \rangle - \langle \Psi_N^v | \Psi_N^v \rangle \qquad (7)$$

The valence nucleon can overlap with more than one core nucleons simultaneously and in addition to six-quark bag can form nine-quark bag, twelve-quark bag etc., thus P_Q^{6q} can be broken up as,

$$P_Q^{6q} = P_{Q_1}^{6q} + P_{Q_2}^{6q} + ... \qquad (8)$$

Where $P_{Q_1}^{6q}$ is the probability that the valence nucleon forms a six-quark bag with only one core nucleon, $P_{Q_2}^{6q}$ refers to the probability that valence nucleon forms a nine-quark bag with any two of the core nucleons and so on [7]. Exact calculation of $P_{Q_1}^{6q}$, $P_{Q_2}^{6q}$ etc., is possible only for three body case. For heavier nuclei, if only the lowest order terms in the expansion of correlation function $\theta(r_0 - r_{ij})$ are retained. The probability $P_{Q_1}^{6q}(r_0)$ that the valence particle forms a six-quark bag with only one core nucleon is,

$$P_{Q_1}^{6q}(r_0) = \langle \psi_{Q_1}^v | \psi_{Q_1}^v \rangle \qquad (9)$$

Where;

$$|\psi_{Q_1}^v\rangle = \phi_{\alpha_0}(\Lambda) A\left\{\sum_{\alpha_i}\theta(r_0 - r_{\alpha_i\alpha_v})\prod_{\alpha_j\neq\alpha_i}^{\alpha_A}\left[1-\theta(r_0 - r_{\alpha_j\alpha_v})\right]\prod_{i=1}^{A+1}\phi_{\alpha_i}(i)\right\} \qquad (10)$$

Similarly the probability $P_{Q_2}^{6q}(r_0)$ that the valence particle forms a nine-quark bag with any two of the core nucleons is

$$P_{Q_2}^{6q}(r_0) = \left\langle \psi_{Q_2}^{v} \middle| \psi_{Q_2}^{v} \right\rangle \tag{11}$$

Where

$$\left.\psi_{Q_2}^{v}\right\rangle = \phi_{\alpha_0}(\Lambda) A \left\{ \sum_{\alpha_i \alpha_j} \theta\left(r_0 - r_{\alpha_i \alpha_v}\right) \theta\left(r_0 - r_{\alpha_i \alpha_v}\right) \prod_{\alpha_m \neq \alpha_j \neq \alpha_i}^{\alpha_A} \left[1 - \theta\left(r_0 - r_{\alpha_m \alpha_v}\right)\right] \prod_{i=1}^{A+1} \phi_{\alpha_i}(i) \right\} \tag{12}$$

The quadratic higher order terms in the correlation function in $P_{Q_1}^{6q}(r_0)$ and $P_{Q_2}^{6q}(r_0)$ can be reduced to first order by using the following identity,

$$\theta\left(r_0 - r_{ij}\right)\theta\left(r_0 - r_{ij}\right) = \theta\left(r_0 - r_{ij}\right) \tag{13}$$

Even then calculation of $P_{Q_1}^{6q}(r_0)$ and $P_{Q_2}^{6q}(r_0)$ etc., for heavier nuclei becomes more and more difficult. If we assume that the chance for the valence particle to overlap with the core particle does not depends upon whether it already overlaps with other core particles, one can calculate $P_{Q_1}^{6q}(r_0)$ and $P_{Q_2}^{6q}(r_0)$ from the average probability $P_{NN}^{6q}(r_0)$ using the following expressions,

$$P_{Q_1}^{6q}(r_0) = P_{NN}^{6q}(r_0)\left(1 - P_{NN}^{6q}(r_0)/A\right)^{A-1} \tag{14}$$

And

$$P_{Q_2}^{6q}(r_0) = \left[\frac{A}{2}\right]\left[\frac{P_{NN}^{6q}(r_0)}{A}\right]^2\left[1 - \frac{P_{NN}^{6q}(r_0)}{A}\right]^{A-2} \tag{15}$$

$P_{NN}^{6q}(r_0)$ can be expressed as a combination of a direct term $p_{n_i l_i j_i}^{d}(r_0)$ and an exchange term $p_{n_i l_i j_i}^{e}(r_0)$ as,

$$P_{NN}^{6q}(r_0) = \sum_{n_i l_i j_i} (2 j_i + 1)\left[2 p_{n_i l_i j_i}^{d}(r_0) - p_{n_i l_i j_i}^{e}(r_0)\right] \tag{16}$$

$P_{n_i l_i j_i t_i}(r_0)$ can be interpreted as the probability for the valence particle to be within a distance r_0 of a specified core particle with quantum numbers $n_i l_i j_i t_i$. The valence nucleon can also overlap with the hyperon and from a six-quark bag with the hyperon as $P_{\Lambda N}^{6q}(r_0)$.The overlap probability $P_{NN}^{6q}(r_0)$ and $P_{\Lambda N}^{6q}(r_0)$ can be estimated either by Moshinsky transformation method [8] or by Slater integral method [9].

In the previous work given by Mehrotra, Mehrotra and Miller [10–12], following problems have been studied in the framework of hybrid quark model (HQM);

a) The quark contribution to the binding energy difference of A=6, 14 mirror hypernuclei.
b) The quark contribution to the magnetic moment of mirror nuclei with closed core + one nucleon.

DISCUSSION

If the six-quark plus NN wavefunction obey the same normalization condition as an ordinary NN wavefunctions, then the six-quark probability equals the probability defect of $\phi_{12}(r)$ for $r < r_0$. This is the simplest prescription for the six-quark probability. But care has to be exercised in choosing the wavefunction $\phi_{12}(r)$. First, because of the different strong dynamics for $r < r_0$, the probability to find six-quark with $r < r_0$ does not have to be the same as that of finding two nucleons at $r < r_0$ in the conventional picture. This change can be accommodated by allowing for a different normalization of the external wavefunction, even though its shape remains the same. Alternatively, the effective potential for $r < r_0$ may have to be modified to accommodate the different dynamics for $r < r_0$. This would lead to a different shape of the external wavefunction. Earlier calculation in nonrelativistic quark model framework indicate that there is no sudden decrease in the six-quark probability for small *r*. Sign change of the s-wave phase shift, which is usually explained by short range repulsion or equivalently by the vanishing of short-range NN wave function, can then be interpreted as the absence of NN components in the short-range six-quark wave function or as a node in the conventional wave function for small *r*. Thus if the short distance behaviour of NN potential is not represented by strong repulsion interaction, then the six-quark probability can be determined as a wavefuction defect of uncorrelated shell model wavefunction.Earlier studies on the continuum [6] and the bound state [13] wavefunction in the two body system show that the current conservation guarantees identity of the six-quark probability and the conventional wavefunction defect for $r < r_0$ as long as we do not change the interaction for $r < r_0$. This is also true for the wave functions obtained with phenomenal NN potential with modest short range repulsion.

CONCLUSION

In the framework of hybrid quark model (HQM);

a) The results of our calculations [10,11] show that the six-quark bag formation effect contributessignificantly to the binding energy difference of mirror hypernuclei pair ${}^{6}_{\Lambda}He \sim {}^{6}_{\Lambda}Li$ and ${}^{14}_{\Lambda}C \sim {}^{14}_{\Lambda}N$.
b) The overlap probability of the valence nucleon with the hyperon also makes a smaller contribution to the binding energy difference and should be included in the reliable calculations.
c) It is also observed that six-quark cluster formation effect increases the binding of Λ -hyperon in the neutron rich partner (${}^{6}_{\Lambda}He, {}^{14}_{\Lambda}C$) compared to that of its proton rich partner (${}^{6}_{\Lambda}Li, {}^{14}_{\Lambda}N$).
d) The correction to the magnetic moment due to the six-quark cluster formation effects as estimated in the previous work [Mehrotra 2017b] [12], makes a sizable contribution to the magnetic moments of mirror nuclei with closed core+one nucleon.

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