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The Square of Rotational Energy and the Moment of Inertia for Pt (A=192 – 198) Isotopes

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Abstract

The aim of the present work was to study the square of rotational energy of Pt (A=192–198) isotopes, depending on the energy levels E(L) and angular momentum (L), as has been the study of moment of inertia for same isotopes, depending on the energy transitions for spin sequences and angular momentum (L). All that has been found by using (IBM-1) program. After that, all of the data from theoretical work were compared with the experimental results and shown through drawing.

Keywords: *Interacting Boson model, square of rotational energy, moment of inertia, Pt isotopes, energy levels*

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INTRODUCTION

One must recourse to approximate to grasp composition of medium- and heavy-mass nuclei. Choosing appropriate approximation depends on the nature of nuclei according to the assumption [1]. Interacting Boson model (IBM) of nuclear structure was suggested by Arima and Iachello [2].

This model represents a significant step forward towards knowledge of nuclear structure. It shows a simple Hamiltonian, and ability to distinguish common nuclear properties through a wide range of the nuclei. It is founded on instead of general algebraic group theoretical techniques. It has also found another application in problems of atomic, molecular and low-energy physics [3]. The application of this model to deformed nuclei is now a subject of much attention.

Pt (A=192–198) isotopes as other isotopes, belong to the region of neutron-rich, A≈190 nuclei, approaching doubly magic, which was marked by varied structure phenomena. While collectivity decreases with increasing neutron number, oblate rotation-aligned states tend to be favored at high spin over prolate excitations [4]; these isotopes have not been as well studied as the lighter ones. These nuclei are particularly interesting for exploring since oblate shapes are predicted to be more favourite. Moreover, because of their transitional nature, both collective and intrinsic degrees of freedom may play an important role in determining the excited level structure.

The region of proton-rich Pt isotopes is characterized by substantial ground-state deformation that decreases in amount for the neutron-rich nuclei; this is mainly due to the existence of N =126 spherical shell gap. Gamma softness in varying degrees is a feature common to all Pt isotopes owing to the relatively small number of valence nucleons [5].

The aim of this work was to study the square of rotational energy and moment of inertia for Pt (A=192–198) isotopes and compare the resulted data from theoretical work with the experimental results.

THEORETICAL BASICS

The Square of Rotational Energy and Moment of Inertia (Backbending Curve)

The collective rotation movement for nucleus relies on the valence nucleons movement with nucleus movement, which causes rotation numeral of nucleons about axis different from nuclear symmetry axis. In some nuclei a revulsion may occur in the value of moment of inertia at the high angular momentum relatively which leads to landing in the rotational energy of the nuclei. These sudden changes lead to happen (curvatures) like letter Z inverses [6].

The formula to calculate all the moment of inertia and the square of rotational energy are [7, 8] given below:

$$\frac{2\vartheta}{\hbar^2} = \frac{4L-2}{E(L)-E(L-2)} = \frac{4L-2}{E_\gamma}(MeV)^{-1} \quad (1)$$

The ϑ is the moment of inertia and E_γ is the transition energy.

$$(\hbar\omega)^2 = \left(\frac{E(L)-(E(L-2)}{\sqrt{L(L+1)}-\sqrt{(L-2)(L-1)}}\right)^2 (MeV)^2 \quad (2)$$

The increase in centrifugal force as a result of increasing the angular velocity of the nucleus (ω) leads to deformation of the nucleus [9]. Figure 1 shows two types of deformation of the rotational nuclei—the first type represents prolate deformation, in this the nucleus revolves around the vertical axis on the nuclear symmetry axis; the second type represents oblate deformation, in this type the nucleus revolves around axis parallel to the nuclear symmetry axis [10]. In both the types when the movement be quick rotational there is a force created called Coriolis force break pairing obtained between the number of nucleons pairs, causing the appearance of (two-quasiparticles) or (four-quasiparticles), causing anomaly at certain angular momentum $L^\pi \geq 10^+$ in some nuclei which cause the (backbending) phenomenon [11].

RESULTS AND DISCUSSION

Ground Energy Bands Spectrum

It was selected ground energy band for even–even Pt (A=192–198) isotopes under study, where these isotopes were classified among the dynamical symmetry SU(5)–O(6) [13]. Note that it has been classified as a dynamical symmetry SU(3)–O(6) [14], but the results are most compatible with the experimental results that have been reached within at most dynamical symmetry SU(5)–O(6).

Table 1 shows the corresponding parameters obtained with the best fitting from the Hamiltonian operator with the convenient dynamical symmetry.

Tables 2–5 shows the energy levels and their transitions of ground state band for even–even Pt (A=192–198) isotopes which were obtained from the program (IBM-1) and their comparison with the experimental values. Some energy levels for certain angular momentum and some of their transitions were predicted, where the predicted energy levels $E(12_1^+)$ for ($^{194}_{78}Pt_{116}$ and $^{196}_{78}Pt_{118}$) isotopes, to be (4.41346 and 4.97680), respectively; and $E(10_1^+)$ for ($^{198}_{78}Pt_{120}$) isotope, to be (4.23488), has also been confirmed. The positive parity of energy level (8_1^+) for ($^{194}_{78}Pt_{116}$) isotope and the energy levels $E(10_1^+)$, $E(8_1^+, 10_2^+)$ for ($^{196}_{78}Pt_{118}$), ($^{198}_{78}Pt_{120}$) isotopes were confirmed, respectively.

In addition, the predicted energy transitions $E_\gamma(12_1^+ - 10_1^+)$ for ($^{194}_{78}Pt_{116}$ and $^{196}_{78}Pt_{118}$) isotopes, to be (1.17556 and 1.34180), respectively; and $E_\gamma(10_1^+ - 8_1^+)$ for ($^{198}_{78}Pt_{120}$) isotope to be (1.32035) were confirmed.

Table 1: *The Parameters Values of Hamiltonian Operator for the ($^{192-198}_{78}Pt$) Isotopes by Using (IBM-1. For) Program [13].*

Isotopes	N	EPS (MeV)	$\hat{P}.\hat{P}$ (MeV)	$\hat{L}.\hat{L}$ (MeV)	$\hat{Q}.\hat{Q}$ (MeV)	$\hat{T}_3.\hat{T}_3$ (MeV)	$\hat{T}_4.\hat{T}_4$ (MeV)	CHI (MeV)	SO(6) (MeV)
$^{192}_{78}Pt_{114}$	8	0.1000	0.1000	0.0150	-0.0150	0.0800	0.0000	0.3000	1.0000
$^{194}_{78}Pt_{116}$	7	0.1000	0.1200	0.0150	-0.0130	0.0900	0.0000	0.3000	1.0000
$^{196}_{78}Pt_{118}$	6	0.1000	0.1300	0.0190	-0.0110	0.0900	0.0000	0.3000	1.0000
$^{198}_{78}Pt_{120}$	5	0.1000	0.1300	0.0230	-0.0070	0.1200	0.0000	0.3000	1.0000

Table 2: *Theoretical Energy Levels and Energy Transitions (g-band) as Compared to the Experimental Data for the $^{192}_{78}Pt_{114}$ Isotope by Using IBM-1.*

L	E(L) (MeV)		Spin sequences $L_f^+ - L_i^+$	E_γ (MeV)	
	EXP. [15, 16, 17]	IBM-1		EXP. [15, 16, 17]	IBM-1
0	0.00000	0.00000	----	----	----
2	0.31651	0.27671	$2_1^+ - 0_1^+$	0.31651	0.27671
4	0.78457	0.73929	$4_1^+ - 2_1^+$	0.46807	0.46258
6	1.36540	1.38213	$6_1^+ - 4_1^+$	0.58083	0.64284
8	2.01837	2.20244	$8_1^+ - 6_1^+$	0.65295	0.82031
10	2.72940	3.19900	$10_1^+ - 8_1^+$	0.71100	0.99656

Table 3: *Theoretical Energy Levels and Energy Transitions (g-band) as Compared to the Experimental Data for the* $^{194}_{78}Pt_{116}$ *Isotope by Using IBM-1.*

L	E(L) (MeV)		Spin sequences $L_f^+ - L_i^+$	E_γ (MeV)	
	EXP. [15, 16, 17]	IBM-1		EXP. [15, 16, 17]	IBM-1
0	0.00000	0.00000	----	----	----
2	0.32846	0.29036	$2_1^+ - 0_1^+$	0.32846	0.29036
4	0.81127	0.76182	$4_1^+ - 2_1^+$	0.48280	0.47146
6	1.41181	1.41125	$6_1^+ - 4_1^+$	0.60050	0.64943
8	2.09953	2.23693	$8_1^+ - 6_1^+$	0.68770	0.82568
10	2.84850	3.23790	$10_1^+ - 8_1^+$	0.74900	1.00097
12	----	4.41346	$12_1^+ - 10_1^+$	----	1.17556

Table 4: *Theoretical Energy Levels and Energy Transitions (g-band) as Compared to the Experimental Data for the* $^{196}_{78}Pt_{118}$ *Isotope by Using IBM-1.*

L	E(L) (MeV)		Spin sequences $L_f^+ - L_i^+$	E_γ (MeV)	
	EXP. [15, 16, 17]	IBM-1		EXP. [15, 16, 17]	IBM-1
0	0.00000	0.00000	----	----	----
2	0.35568	0.31294	$2_1^+ - 0_1^+$	0.35568	0.31294
4	0.87686	0.83458	$4_1^+ - 2_1^+$	0.52117	0.52164
6	1.52580	1.56298	$6_1^+ - 4_1^+$	0.64930	0.72840
8	2.25270	2.49678	$8_1^+ - 6_1^+$	0.72740	0.93380
10	3.04400	3.63500	$10_1^+ - 8_1^+$	0.79130	1.13822
12	----	4.97680	$12_1^+ - 10_1^+$	----	1.34180

Table 5: *Theoretical Energy Levels and Energy Transitions (g-band) as Compared to the Experimental Data for the* $^{198}_{78}Pt_{120}$ *Isotope by Using IBM-1.*

L	E(L) (MeV)		Spin sequences $L_f^+ - L_i^+$	E_γ (MeV)	
	EXP. [15, 16, 17]	IBM-1		EXP. [15, 16, 17]	IBM-1
0	0.00000	0.00000	----	----	----
2	0.40722	0.37088	$2_1^+ - 0_1^+$	0.40721	0.37088
4	0.98507	0.98123	$4_1^+ - 2_1^+$	0.57782	0.61035
6	1.71417	1.82957	$6_1^+ - 4_1^+$	0.72910	0.84834
8	2.52710	2.91453	$8_1^+ - 6_1^+$	0.81300	1.08496
10	----	4.23488	$10_1^+ - 8_1^+$	----	1.32035

Table 6: *Theoretical Energy Ratios Values* $E(4_1^+)/E(2_1^+)$, $E(6_1^+)/E(2_1^+)$, $E(8_1^+)/E(2_1^+)$ *and their Comparison with the Experimental Data According to the (IBM-1) for the* ($^{192-198}_{78}Pt$) *Isotopes.*

Isotope	$E(4_1^+)/E(2_1^+)$		$E(6_1^+)/E(2_1^+)$		$E(8_1^+)/E(2_1^+)$	
	EXP. [15, 16, 17]	IBM-1	EXP. [15, 16, 17]	IBM-1	EXP. [15, 16, 17]	IBM-1
$^{192}_{78}Pt_{114}$	2.47886	2.67171	4.31397	4.99487	6.37703	7.95938
$^{194}_{78}Pt_{116}$	2.46989	2.62371	4.29822	4.86034	6.39196	7.70399
$^{196}_{78}Pt_{118}$	2.46529	2.66690	4.28976	4.99450	6.33343	7.97846
$^{198}_{78}Pt_{120}$	2.41901	2.64568	4.20944	4.93305	6.20574	7.85842

The results have been used for calculating the energy ratios and compared them with the identical and available experimental values as shown in Table 6.

The Square of Rotational Energy and the Moment of Inertia

The rotational motion is very complicated because it represents the rugged body rotation for each rotation of deformed surface including N of free particles.

The rotational motion leads to the formation of large quadrupole deformation for states of excited low-lying nuclei and the rotation should not be a rugged body rotation. The spherical surface of the nucleus will rotate and rotational levels for all nucleons will rotate too. Table 7.

Large number of nuclear energy levels has been distinguished as rotational levels, because it gives close spectrums to the typical spectra for determining rotational levels. Table 8

Table 7: *Comparison Between the IBM-1 and Experimental Values for Square of Rotational Energy & Moment of Inertia for Pt (A=192) Isotope.*

L	$\hbar^2\omega^2$		$\frac{2\vartheta}{\hbar^2}$	
	EXP. [15, 16, 17]	IBM-1	EXP. [15, 16, 17]	IBM-1
2	0.01670	0.01276	18.95697	21.68335
4	0.05355	0.05230	29.91006	30.26503
6	0.08362	0.10243	37.87683	34.22313
8	0.10611	0.16746	45.94532	36.57154
10	0.12604	0.24759	53.44585	38.13117

Table 8: *Comparison Between the IBM-1 and Experimental Values for Square of Rotational Energy & Moment of Inertia for Pt (A=194) Isotope.*

L	$\hbar^2\omega^2$		$\frac{2\vartheta}{\hbar^2}$	
	EXP. [15, 16, 17]	IBM-1	EXP. [15, 16, 17]	IBM-1
2	0.01798	0.01405	18.26701	20.66400
4	0.05698	0.05433	28.99769	29.69499
6	0.08939	0.10454	36.63614	33.87586
8	0.11770	0.16966	43.62367	36.33369
10	0.13985	0.24978	50.73431	37.96317
12	----	0.34483	----	39.13029

Table 9: *Comparison Between the IBM-1 and Experimental Values for Square of Rotational Energy & Moment of Inertia for Pt (A=196) Isotope.*

L	$\hbar^2\omega^2$		$\frac{2\vartheta}{\hbar^2}$	
	EXP. [15, 16, 17]	IBM-1	EXP. [15, 16, 17]	IBM-1
2	0.02108	0.01632	16.86891	19.17300
4	0.06639	0.06651	26.86238	26.83843
6	0.10438	0.13151	33.88264	30.20318
8	0.13150	0.21701	41.24278	32.12679
10	0.15610	0.32298	48.02224	33.38546
12	----	0.44925	----	34.28231

Table 10: *Comparison Between the IBM-1 and Experimental Values for Square of Rotational Energy & Moment of Inertia for Pt (A=198) Isotope.*

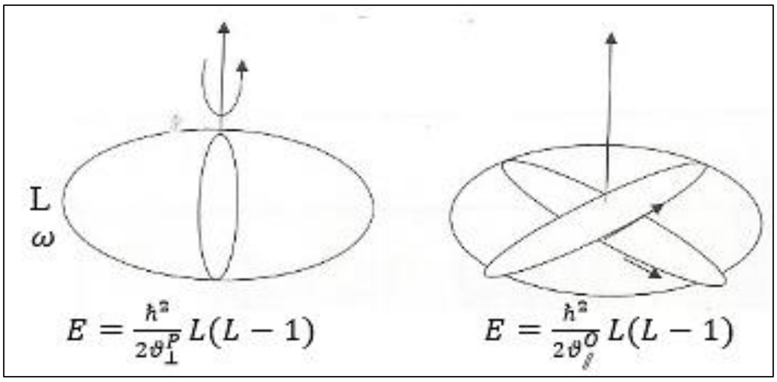
L	$\hbar^2\omega^2$		$\frac{2\vartheta}{\hbar^2}$	
	EXP. [15, 16, 17]	IBM-1	EXP. [15, 16, 17]	IBM-1
2	0.02764	0.02292	14.73441	16.17774
4	0.08162	0.09106	24.22900	22.93766
6	0.13176	0.17838	30.17419	25.93300
8	0.16447	0.29295	36.90037	27.65079
10	----	0.43461	----	28.78025

In all cases it was found that the moment of inertia, which must be used to obtain agreement with the experimental result, is less from the moment of inertia of the body hard by a factor of 2 to 4 as the nucleons drift with the surface during the rotational motion. It has been found experimentally that the moment of inertia increases as the deformation constant β increases. Table 9 and Table 10

In some nuclei occurs a sudden change in the value of moment of inertia at the high angular momentum that relatively leads to landing in the rotational energy of the nuclei. These sudden changes lead to occur (curvatures) in the field of energy with angular momentum.

Figure 2 shows the comparison between theoretical and experimental results of square of rotational energy as a function of angular momentum for Pt (A=192–198) isotopes, note that good agreement between this results.

Figure 3 shows the moment of inertia as a function of angular momentum (theoretical and experimental) but there recursive difference increases with increasing excited energy. This difference can be explained as a result of increased moment of inertia with increase in angular momentum of the rotation, and that the effect of centrifugal force. When we draw the relationship between moment of inertia $\frac{2\vartheta}{\hbar^2}$ and the square of rotational energy $\hbar^2\omega^2$ the change clearly shows the shape of the letter Z inverses. After finding the energy levels using IBM-1 and angular moment to the yrast energy levels, the square of rotational energy and the moment of inertia can be calculated from Eqs. (1–2).



Rotation of a prolate nucleus *Rotation of an oblate nucleus*

Fig. 1: *The Rotational Motion for Prolate and Oblate Nuclei, Where $\vartheta_\perp^P$ and $\vartheta_{//}^O$ Represents the Moment of Inertia for Two Type, Respectively [12].*

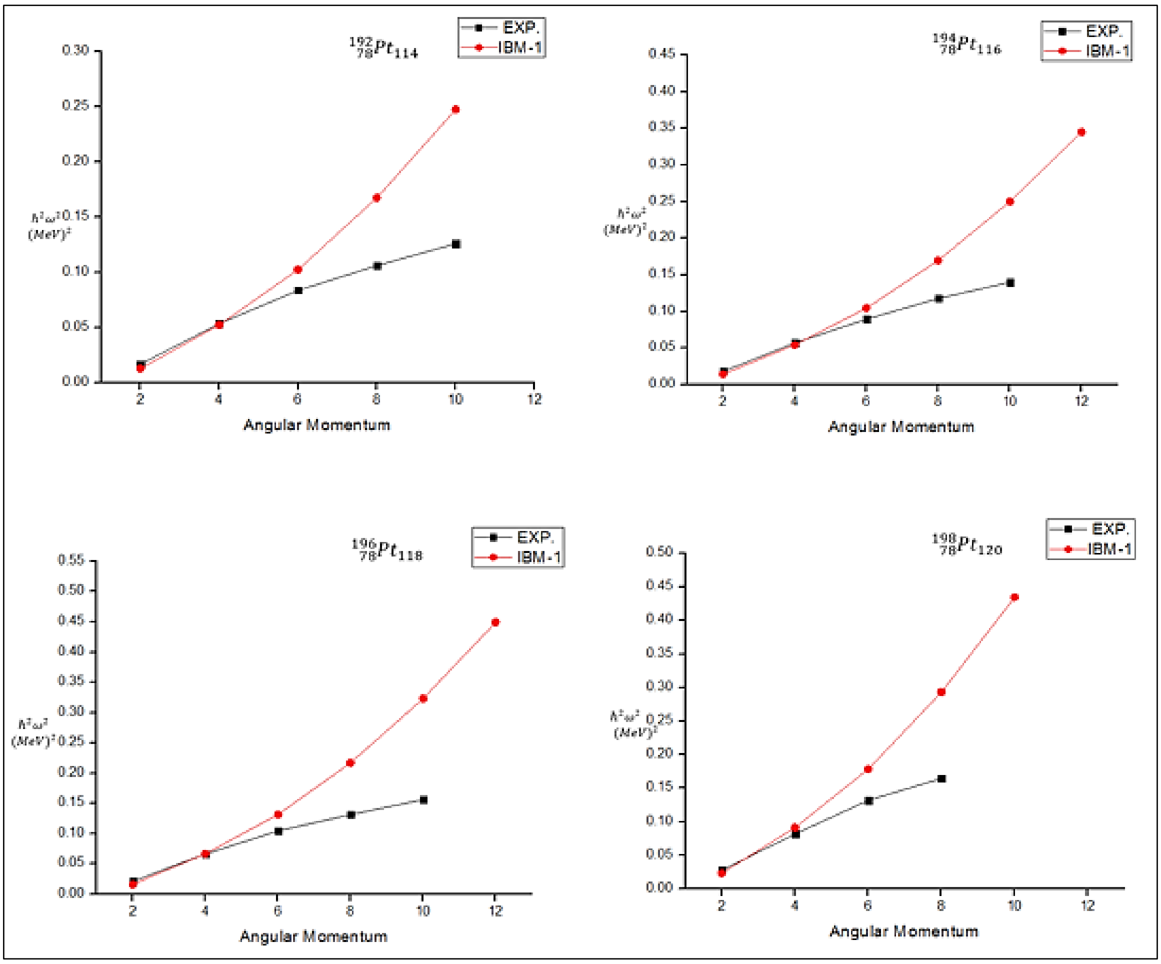


Fig. 2: *The Relation Between $\hbar^2\omega^2$ as a Function of Angular Momentum for Pt (A=192–198) Isotopes.*

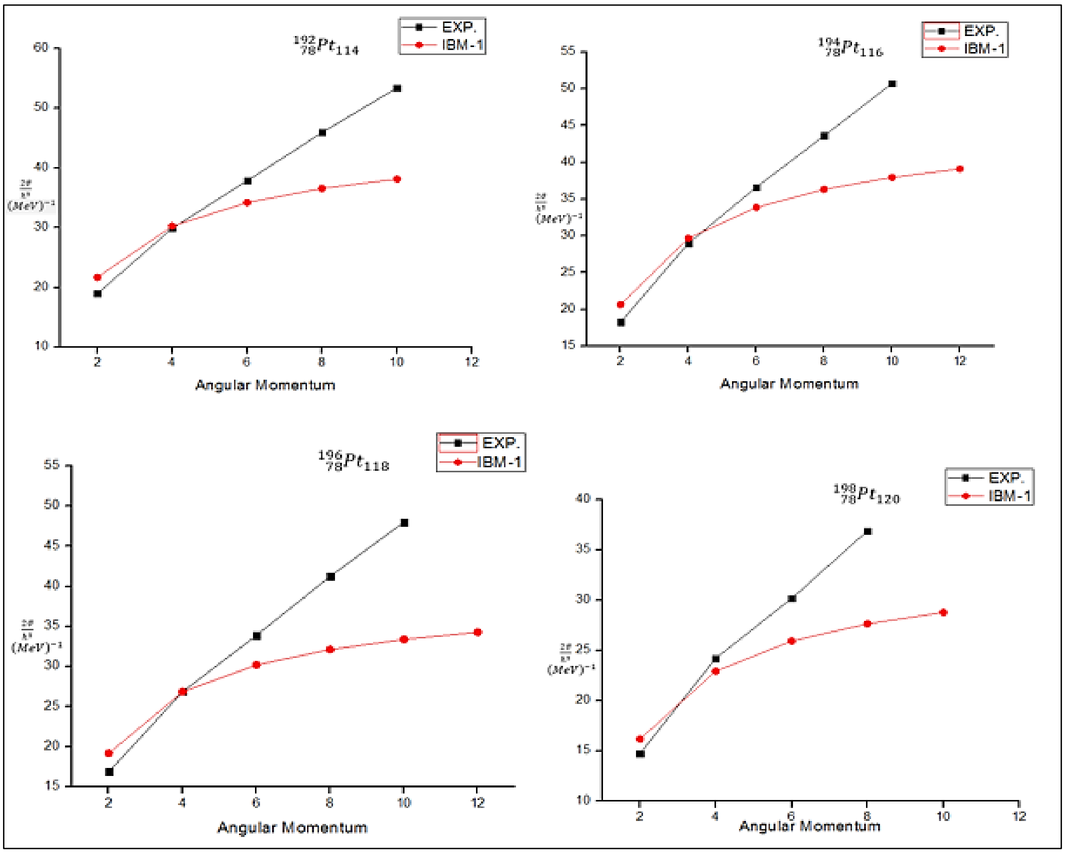


***Fig. 3:** The Relation Between* $\frac{2\vartheta}{\hbar^2}$ *as a Function of Angular Momentum for Pt (A=192–198) Isotopes.*

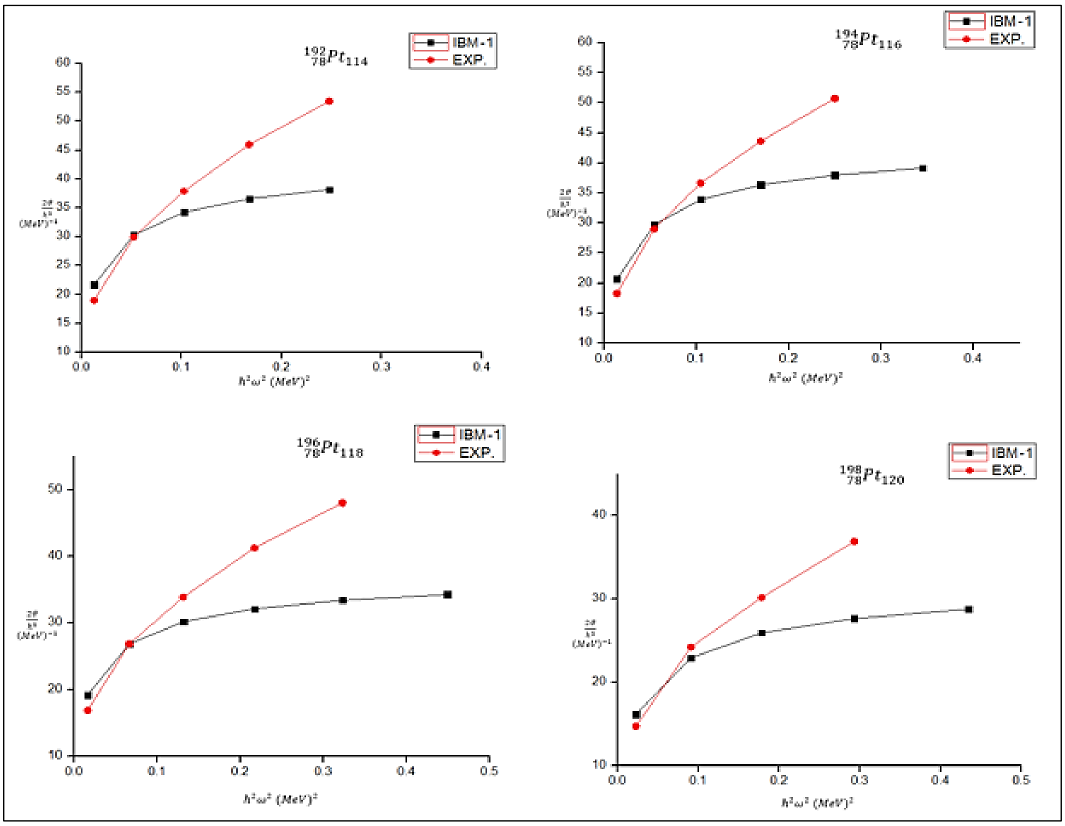


***Fig. 4:** The Calculated and Observed Moment of Inertia* $\frac{2\vartheta}{\hbar^2}$ *vs.* $\hbar^2\omega^2$ *for yrast Levels of Pt (A=192-198) Isotopes. The Experimental Data were Taken From [15–17].*

Figure 4 shows the relation of the moment of inertia $\frac{2\vartheta}{\hbar^2}$ as a function of the square of the energy $\hbar^2\omega^2$ (backbending curve) of the emitted photon for all Pt isotopes under study. When the nuclei transform from the (L) state to the (L-2) state, the even-even Pt isotopes do not suffer from any curvature which indicates the absence of change in their properties. The comparison between theoretical and experimental values [15–17] for all square rotational energy and the moment of inertia are shown in Figure 4.

CONCLUSIONS

From this research, following was concluded:

- Some energy levels for certain angular momentum and some of their transitions were predicted.
- Square of rotational energy and the moment of inertia of Pt (A=192–198) isotopes was calculated to contribute to reaching the basic concepts of nuclear structure.
- The shape of this nuclei was determined through the drawing as functions for angular momentum.

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