

# Kantowski-Sachs Cosmological Model with Quadratic Equation of State in $f(T)$ Gravity

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## Abstract

In this paper, we have investigated Kantowski-Sachs cosmological model with Quadratic Equation of State (QEoS) in the framework of Teleparallel Gravity, so called  $f(T)$  gravity, where  $T$  denotes the torsion scalar. The behavior of accelerating universe is discussed by considering Linear  $f(T)$  gravity i.e.  $f(T) = \alpha T + \beta$ . The solutions of field equations are obtained by using variable deceleration parameter. The physical behavior of these models has been discussed using some physical quantities. Also, the function of the torsion scalar for the universe is evaluated.

**Keywords:** Kantowski-Sachs universe, QEoS,  $f(T)$  gravity, VDP

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## INTRODUCTION

Recent cosmological observations such as the type-Ia Supernova, the Cosmic Microwave Background Radiation (CMBR), the Large Scale Structure (LSS) and Wilkinson Microwave Anisotropy Probe (WMAP) indicate that the universe is spatially flat and consists of about 70% of an exotic energy which has a component with negative pressure so called dark energy (DE) (whose nature and cosmological origin still remain enigmatic at present) which indicates that our universe is undergoing in accelerating expansion phase [1–5]. The simplest candidate of the DE is cosmological constant  $\Lambda$  but there are serious theoretical problems (fine-tuning, coincidence and so on) associated with it. The other alternative approach dealing with the acceleration problem of the universe is changing the gravity law through the modification of action in General Theory of Relativity (GTR). Various modifications in the action of GTR are present, out of which, one

replaces the Ricci scalar  $R$  in the Einstein-Hilbert action by an arbitrary function of  $R$  belongs to the well-known  $f(R)$  modified gravity. Originate from the exact solution with power law  $f(R)$  cosmological model, Capozziello *et al.* achieved dust matter and DE phase [6]. Using the same theory, Azadi *et al.* studied vacuum solution in cylindrically symmetric space time [7]. Bianchi type-III, cosmological models with bulk viscosity in  $f(R)$  theory were investigated by Katore and Shaikh [8]. Miranda *et al.* discussed a viable singularity-free  $f(R)$  gravity without a cosmological constant [9]. Sharif and Yousaf studied the impact of DE and dark matter models on the dynamical evolution of collapsing self-gravitating systems in this gravity [10]. Bhoyar *et al.* examined Bianchi type-I space-time with quadratic equation of state in  $f(R)$  gravity [11]. Chirde and Shekh discussed isotropic background for interacting two fluid scenarios

coupled with zero mass scalar fields in this gravity [12]. Recently, the same authors Chirde and Shekh gave the solution of quadratic equation of state with constant deceleration parameter in  $f(R)$  gravity [13].

Second is the gravitational action which includes an arbitrary function of the Ricci scalar and trace of the stress energy tensor known as  $f(R, T)$  gravity. Several authors have investigated the aspect of cosmological models in this gravity [14–17]. Recently, Chirde and Shekh looked into a non-static plane symmetric space-time filled DE within the frame work of  $f(R, T)$  gravity [18]. Bhoyar *et al.* talked about non-static plane symmetric cosmological model with magnetized anisotropic DE by hybrid expansion law and thermo dynamical aspects of non-static plane symmetric Universe in  $f(R, T)$  gravity [19, 20]. Very recently, Chirde and Shekh discussed plane symmetric dark energy models in the form of wet dark fluid in  $f(R, T)$  gravity [21].

Among the various modifications of Einstein’s theory, another one way to look at the theory beyond GTR is the Teleparallel Gravity (TG) which is different from GTR (i.e. uses the Weitzenbock connection in place of the Levi-Civita connection) which has no curvature but has torsion, responsible for the acceleration of the universe. Bamba *et al.* discussed the EoS parameter for exponential, logarithmic as well as combination of these  $f(T)$  models and they concluded that the crossing of phantom divide line is observed in combined model only [22]. The graphical representation of k-essence in this modified gravity with the help of EoS parameter was described by Sharif and Rani [23]. Spherically symmetric solutions in  $f(T)$  gravity were obtained by Wang [24], in  $f(T)$  gravity, the existence of relativistic stars was investigated by Bohmer *et al.* [25]. In this gravity, Chirde and Shekh have discussed some cosmological models with different source [26–28]. Recently, Bhoyar *et al.* discussed stability of accelerating universe with linear equation of state in  $f(t)$  gravity using hybrid expansion law [29].

An important theoretical astronomic observation by Misner in 1968 and the modern experimental data have revealed that on large scale, the universe is isotropic and homogeneous in its present state of evolution but it might not be the same in the past. Therefore the models with anisotropic background that approach to isotropy at late times are most suitable for describing the entire evolution of the universe. The spatially homogeneous and anisotropic Kantowski-Sachs space-time provides such a framework, along with, it has astrophysical importance because they are considered as possible candidates for an early era in cosmology. Hence the authors have become interested to investigate the features of Kantowski-Sachs space-time in the recent paper.

Incited by the above discussions in this paper, we explore the Kantowski-Sachs space-time with perfect fluid quadratic equation of state in  $f(T)$  gravity. The paper is organized as follows: In the next part of the paper, we have given a field equation for  $f(T)$  gravity. the following part contains field equations and their solutions. Finally, the last part contains a concluding remark.

### $f(T)$ GRAVITY FORMALISM

In this section, we give a brief description of the  $f(T)$  model and a detailed derivation of its field equations.

Let us define the notations of the Latin subscripts as these are related to the tetrad field and the Greek one is related to the space-time coordinates. For a general space-time metric, we can define the line element as:

$$dS^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{1}$$

This line element can be converted to the Minkowski’s description of the transformation called tetrad, as follows:

$$dS^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij} \theta^i \theta^j, \tag{2}$$

$$dx^\mu = e_i^\mu \theta^i, \quad \theta^i = e_\mu^i dx^\mu, \tag{3}$$

Where,  $\eta_{ij}$  is a metric on Minkowski space-time and  $\eta_{ij} = \text{diag}[1, -1, -1, -1]$  and  $e_i^\mu e_\nu^i = \delta_\nu^\mu$

or  $e_i^\mu e_\mu^j = \delta_i^j$ . The root of metric determinant is given by  $\sqrt{-g} = \det[e_\mu^i] = e$ . For a manifold in which the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) and only the non-zero torsion terms exist, the Weitzenbocks connection components are defined as:

$$\Gamma_{\mu\nu}^\alpha = e_i^\alpha \partial_\nu e_\mu^i = -e_\mu^i \partial_\nu e_i^\alpha \quad (4)$$

which has a zero curvature but nonzero torsion. Through the connection, we can define the components of the torsion tensors as:

$$T_{\mu\nu}^\alpha = \Gamma_{\mu\nu}^\alpha - \Gamma_{\nu\mu}^\alpha = e_i^\alpha (\partial_\mu e_\nu^i - \partial_\nu e_\mu^i) \quad (5)$$

The difference between the Levi-Civita and Weitzenbock connections is a space-time tensor, and is known as the contorsion tensor:

$$K_\alpha^{\mu\nu} = \left(-\frac{1}{2}\right) (T^{\mu\nu}{}_\alpha + T^{\nu\mu}{}_\alpha - T_\alpha^{\mu\nu}) \quad (6)$$

For facilitating the description of the Lagrangian and the equations of motion, we can define another tensor  $S_\alpha^{\mu\nu}$  from the components of the torsion and contorsion tensors, as:

$$S_\alpha^{\mu\nu} = \left(\frac{1}{2}\right) (K^{\mu\nu}{}_\alpha + \delta_\alpha^\mu T^{\beta\nu}{}_\beta - \delta_\alpha^\nu T^{\beta\mu}{}_\beta) \quad (7)$$

The torsion scalar  $T$  is:

$$T = T_{\mu\nu}^\alpha S_\alpha^{\mu\nu} \quad (8)$$

Now, we define the action by generalizing the TG i.e.  $f(T)$  theory as [26]:

$$S = \int [T + f(T) + L_{matter}] e d^4x \quad (9)$$

Here,  $f(T)$  denotes an algebraic function of the torsion scalar  $T$ . Making the functional variation of the action Eq. (9) with respect to the tetrads, we get the following equations of motion:

$$S_\mu^{\nu\rho} \partial_\rho T f_T + \left[ e^{-1} e_\mu^i \partial_\rho \left( e e_i^\alpha S_\alpha^{\nu\rho} \right) + T^\alpha{}_{\lambda\mu} S_\alpha^{\nu\lambda} \right] (1 + f_T) + \frac{1}{4} \delta_\mu^\nu (T + f) = 4\pi T_\mu^\nu \quad (10)$$

The field Eq. (10) is written in terms of the tetrad and partial derivatives and appears very different from Einstein's equations.

Where,  $T_\mu^\nu$  is the energy momentum tensor,  $f_T = df(T)/dT$  and by setting

$f(T) = a_0 = \text{constant}$ , this is dynamically equivalent to the GR.

## FIELD EQUATIONS AND THEIR SOLUTIONS

In this section, we find the solutions for Kantowski-Sachs space-time in  $f(T)$  gravity and some physical quantities.

### Field Equations and Some Physical Quantities

The line element of Kantowski-Sachs space-time is given by:

$$ds^2 = dt^2 - A^2 dr^2 - B^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (11)$$

Where, the metric potentials  $A$  and  $B$  be the functions of time  $t$  only.

Using the space-time Eq. (11), Shekh *et al.* have investigated Kantowski-Sachs cosmological model with perfect fluid in the framework of exponential Teleparallel Gravity and obtained a big-bang singular model which starts its expansion from zero volume with infinite energy density and pressure and it continues to expand with time, it also suggests that the expansion after a large time will stop completely and the universe will achieve isotropy [30].

The corresponding Torsion scalar is given by:

$$T = -2 \left( 2 \frac{\dot{A} \dot{B}}{A B} + \frac{\dot{B}^2}{B^2} \right) \quad (12)$$

Let us consider that the matter content is a perfect fluid such that the energy momentum tensor  $T_\mu^\nu$  is taken as:

$$T_\mu^\nu = (p + \rho) u^\nu u_\mu - p g_\mu^\nu \quad (13)$$

Satisfying the equation of state:

$$p = \varepsilon \rho^2 - \rho \quad (14)$$

(Here, we have assumed an equation of state (EoS) in the general form  $p = p(\rho)$  for the matter distribution)

Where,  $\varepsilon$  is the constant and strictly  $\varepsilon \neq 0$ .

Together with comoving coordinates:

$$u^\nu = (0, 0, 0, 1) \text{ and } u^\nu u_\nu = 1, \quad (15)$$

Where,  $u^\nu$  is the four-velocity vector of the fluid,  $p$  and  $\rho$  be the pressure and energy density of the fluid respectively.

The Kantowski-Sachs space-time Eq. (11), for the fluid of stress energy tensor Eq. (13), and the equation of motion Eq. (10), can be written as:

$$(T + f) + 4(1 + f_T) \left\{ \frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A} \dot{B}}{A B} \right\}, \quad (16)$$

$$+ 4 \frac{\dot{B}}{B} \dot{T} f_{TT} = -16\pi(\varphi^2 - \rho)$$

$$(T + f) + 2(1 + f_T) \left\{ \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 3 \frac{\dot{A} \dot{B}}{A B} \right\} \quad (17)$$

$$+ 2 \left\{ \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right\} \dot{T} f_{TT} = -16\pi(\varphi^2 - \rho)$$

$$(T + f) + 4(1 + f_T) \left\{ \frac{\dot{B}^2}{B^2} + 2 \frac{\dot{A} \dot{B}}{A B} \right\} = 16\pi(\rho). \quad (18)$$

Where, the dot( $\cdot$ ) denotes the derivative with respect to time  $t$ .

Finally, here we have three differential equations with five unknowns namely  $A, B, f, p, \rho$ . In the following part, we define some physical quantities of the Universe.

We define average scale factor and volume respectively as:

$$a = \sqrt[3]{AB^2}, \quad V = a^3. \quad (19)$$

The Hubble's parameter is:

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \quad (20)$$

Where,  $H_1, H_2, H_3$  are the directional Hubble parameters in the direction of  $x, y,$  and  $z$ -axis respectively.

Using Eqs. (19) and (20), we obtain:

$$H = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3} (H_1 + H_2 + H_3) = \frac{\dot{a}}{a}. \quad (21)$$

The mean anisotropy parameter is given by:

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i - H}{H} \right)^2. \quad (22)$$

The expansion scalar and shear scalar are defined as follows:

$$\theta = u^\mu{}_{;\mu} = \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B}, \quad (23)$$

$$\sigma^2 = \frac{3}{2} H^2 \Delta. \quad (24)$$

To find the deterministic solution of the field

equation, some additional conditions are necessary.

i) First we have assumed that the expansion scalar is proportional to the shear scalar which gives us:

$$A = B^n \quad (25)$$

Where,  $n$  be any real proportionality constant.

The motive behind assuming the above relation is that the observations of the velocity red-shift relation for extra galactic sources suggest that Hubble expansion of the universe is isotropy today within  $\approx 30\%$ . To put more precisely, red-shift studies place the limit  $(\sigma/H) \leq 0.3$  on the ratio of shear  $\sigma$  to Hubble constant  $H$  in the neighborhood of our galaxy today. Collin *et al.* have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition  $(\sigma/\theta)$  is constant [31].

For any physically relevant model, Hubble parameter (which provides the volumetric expansion rate of the universe) and deceleration parameter (DP) (which tells whether the universe exhibits accelerating volumetric expansion or not) are the most important observational quantities in cosmology (Figure 5). During 1960s and 1970s, redshift magnitude test has had a chequered history and drew very categorical conclusions. The DP  $q$  lies between 0 and 1, thus it was claimed that the universe is decelerating [32]. Berman [33], Berman and Gomide [34] have proposed a law of variation for Hubble parameter that yields a constant value of DP and get value  $q \geq -1$ , but  $-1 \leq q \leq 0$  corresponds to accelerating expansion. According to the recent theoretical analysis of SNe-Ia, LSS, and CMB, the Universe is spatially flat and has a phase transition i.e. past deceleration to recent acceleration. So, in order to match the results with this observation, many authors have defined different types of solutions (corresponds to DP and scale factor).

Singh and Debnath had defined a special form of DP which is linear in time with a negative slope as [35]:

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{\alpha}{1 + a^\alpha}, \quad (26)$$

Where,  $\alpha > 0$  is a constant and  $a$  is mean scale factor of the universe.

From Eq. (26), we observed that when  $a = 0$ ,  $q = \alpha - 1 > 0$ ,  $q = 0$  for  $a^\alpha = \alpha - 1$ , and for  $a^\alpha > \alpha - 1$ ,  $q < 0$ . We assume that  $a = 0$  for  $t = 0$ , therefore, the universe begins with a decelerating expansion and the expansion changes from past decelerating phase to recent accelerating one. This cosmological scenario is in agreement with theoretical observations. After solving Eq. (26), one can obtain the scale factor of the form as:

$$a = \left( e^{\alpha kt} - 1 \right)^{\frac{1}{\alpha}} \quad (27)$$

For the mean scale factor Eq. (27), we get the value of the DP as:

$$q = \frac{\alpha}{e^{\alpha kt}} - 1. \quad (28)$$

Eq. (28) designates that the DP comes out to be time dependent. At initial epoch ( $t = 0$ ) when universe starts to expand,  $q = \alpha - 1 (> 0)$  for  $\alpha > 1$ , the sign of  $q$  becomes positive which corresponds to the standard decelerating expansion, with the expansion of the universe, the sign of  $q$  become negative which corresponds to the standard accelerating expansion. This scenario is consistent with recent theoretical observations (Figure 1)

$$A = \left( e^{\alpha kt} - 1 \right)^{\frac{3n}{\alpha(n+2)}}, \quad (29)$$

$$B = \left( e^{\alpha kt} - 1 \right)^{\frac{3}{\alpha(n+2)}}. \quad (30)$$

Using the values of Eqs. (29) and (30) we get:

$$ds^2 = dt^2 - \left( e^{\alpha kt} - 1 \right)^{\frac{6n}{\alpha(n+2)}} dx^2 - \left( e^{\alpha kt} - 1 \right)^{\frac{6}{\alpha(n+2)}} (dy^2 + dz^2).$$

The Torsion scalar  $T$  becomes:

$$T = \frac{-18k^2(2n+1)}{(n+2)^2} \left\{ \frac{1}{\left( 1 - e^{-\alpha kt} \right)^2} \right\} \quad (31)$$

The spatial volume becomes:

$$V = \left( e^{\alpha kt} - 1 \right)^{\frac{3}{\alpha}}. \quad (32)$$

The mean Hubble parameter and the expansion scalar comes out to be:

$$H = \frac{k}{\left( 1 - e^{-\alpha kt} \right)}. \quad (33)$$

$$\theta = \frac{3k}{\left( 1 - e^{-\alpha kt} \right)}, \quad (34)$$

The mean anisotropy parameter and shear scalar are given by:

$$\Delta = -1 + \frac{3(n^2 + 2)}{(n+2)^2}, \quad (35)$$

$$\sigma^2 = \frac{3}{2} \left\{ -1 + \frac{3(n^2 + 2)}{(n+2)^2} \right\} \frac{k^2}{\left( 1 - e^{-\alpha kt} \right)^2} \quad (36)$$

The spatial volume of the universe starts with big bang at  $t = 0$  and with the increase of time it always expands, when  $t = \infty$ , then spatial volume  $V \rightarrow \infty$ . Hubble's parameter, expansion scalar and shear scalar, initially all are infinitely large but with the expansion of the universe, these parameters decrease. The behavior is shown in Figure 2.

Thus, energy density and pressure of the universe becomes:

$$\rho = \frac{1}{\chi^2} \left\{ \frac{36k^2(2n+1)}{(n+2)^2} \right\} \frac{1}{\left( 1 - e^{-\alpha kt} \right)^2}, \quad (37)$$

$$p = \frac{36k^2(2n+1)}{(n+2)^2} \frac{1}{\left( 1 - e^{-\alpha kt} \right)^2} \left\{ \frac{36\alpha k^2(2n+1)}{(n+2)^2 \left( 1 - e^{-\alpha kt} \right)^2} - 1 \right\} \quad (38)$$

From Eq. (37), it is observed that the energy density is a decreasing function of time  $t$  and with the expansion of the universe, it converges to positive constant value.

### Model-II

We consider the value of the average scale factor corresponding to the model of the universe as:

$$a = te^t. \quad (39)$$

For the mean scale factor Eq. (39), we get the value of the deceleration parameter as:

$$q = -1 + \frac{1}{(t+1)^2}. \tag{40}$$

$$A_m = \frac{2(n-1)^2}{(n+2)^2}, \tag{47}$$

For this model, the corresponding metric coefficients  $A$  and  $B$  comes out to be:

$$A = (te^t)^{\frac{3n}{(n+2)}}, \tag{41}$$

$$B = (te^t)^{\frac{3}{(n+2)}}. \tag{42}$$

Using the Eqs. (41) and (42), we get:

$$ds^2 = dt^2 - (te^t)^{\frac{6n}{(n+2)}} dx^2 - (te^t)^{\frac{6}{(n+2)}} (dy^2 + dz^2)$$

The Torsion scalar becomes:

$$T = \frac{-18(2n+1)}{(n+2)^2} \left\{ 1 + \frac{1}{t} \right\}^2, \tag{43}$$

The spatial volume becomes:

$$V = t^3 e^{3t} \tag{44}$$

The mean Hubble parameter and the expansion scalar turn out to be:

$$H = \left( 1 + \frac{1}{t} \right), \tag{45}$$

$$\theta = 3 \left( 1 + \frac{1}{t} \right), \tag{46}$$

The mean anisotropy parameter and shear scalar are given by:

$$\sigma^2 = \frac{3(n-1)^2}{(n+2)^2} \left( 1 + \frac{1}{t} \right)^2. \tag{48}$$

In this model, initial time  $t=0$ , the spatial volume vanishes and expands exponentially as time increases and becomes infinitely large at  $t=\infty$ . The mean anisotropy parameter is independent on time  $t$  and constant throughout the evolution of the universe from early to infinite expansion but shear scalar is time dependent and decreases with time. The universe expands with time but the rate of expansion decreases to constant value, which shows that the universe starts evolving with an infinite rate of expansion (Figure 4).

Energy density and pressure of the universe become:

$$\rho = \frac{1}{\chi^2} \left\{ \frac{18(2n+1)}{(n+2)^2} \left( 1 + \frac{1}{t} \right)^2 \right\}, \tag{49}$$

$$p = \frac{18(2n+1)}{(n+2)^2} \left( 1 + \frac{1}{t} \right)^2 \left\{ \frac{18\varepsilon(2n+1)}{(n+2)^2} \left( 1 + \frac{1}{t} \right)^2 - 1 \right\} \tag{50}$$

The energy density is positive and showing the same behavior as that of Figure 3 i.e. at an initial stage, it is high but with the expansion it is gradually decreases up to constant value.

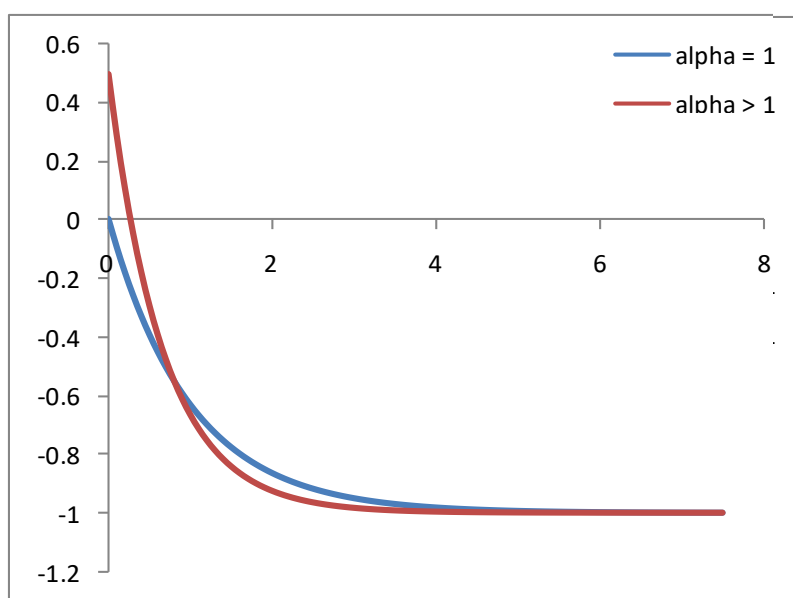
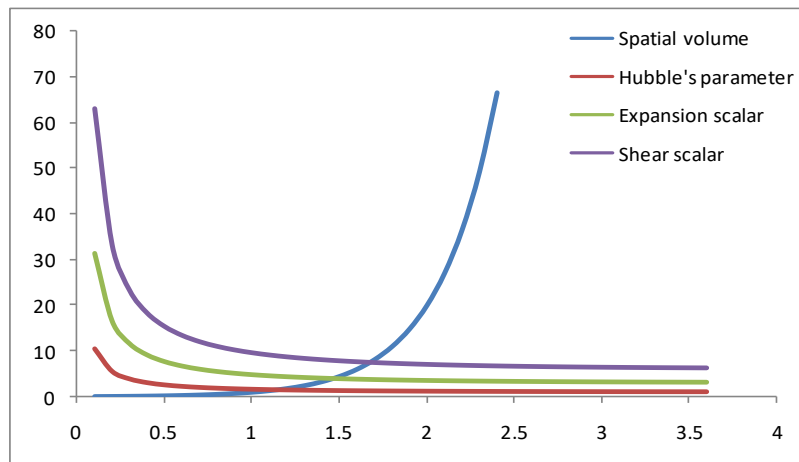
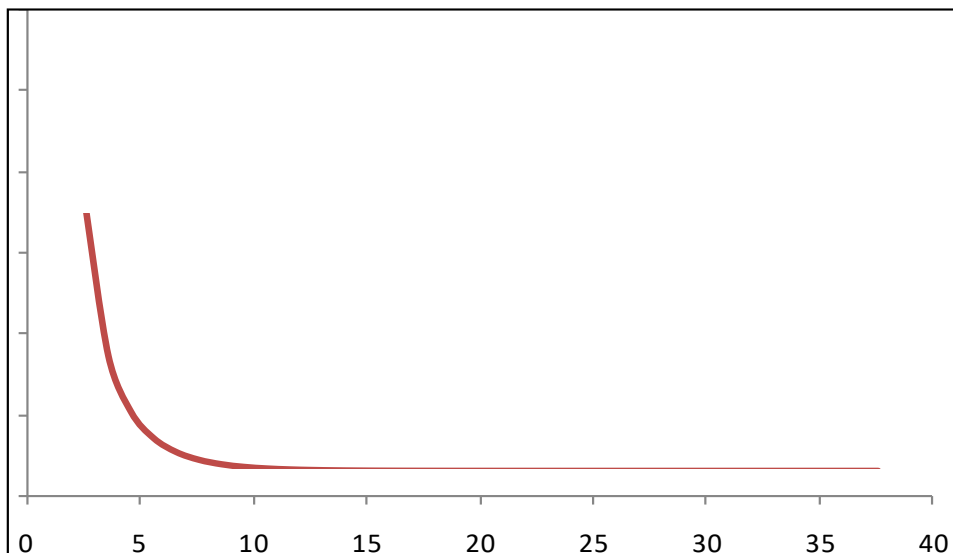


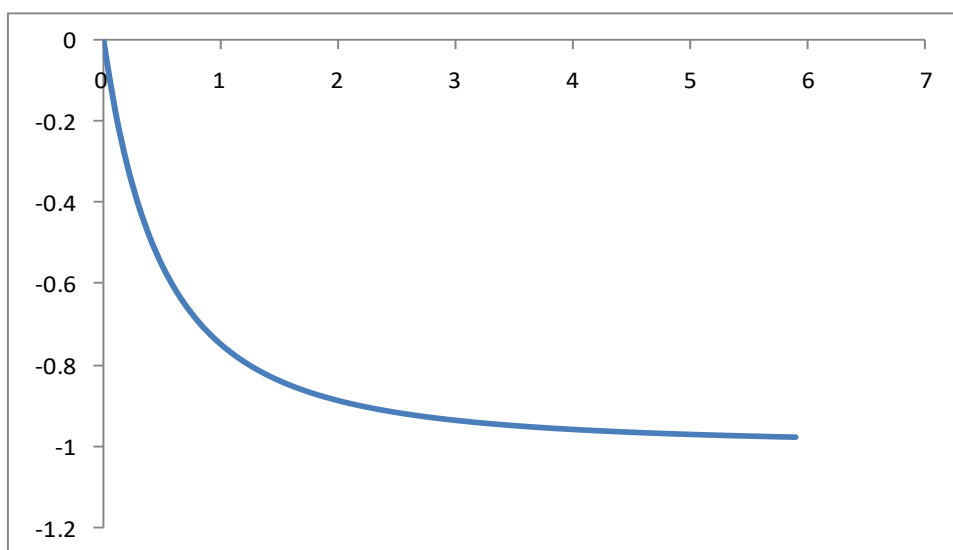
Fig. 1: Behavior of Deceleration Parameter versus Time  $t$ .



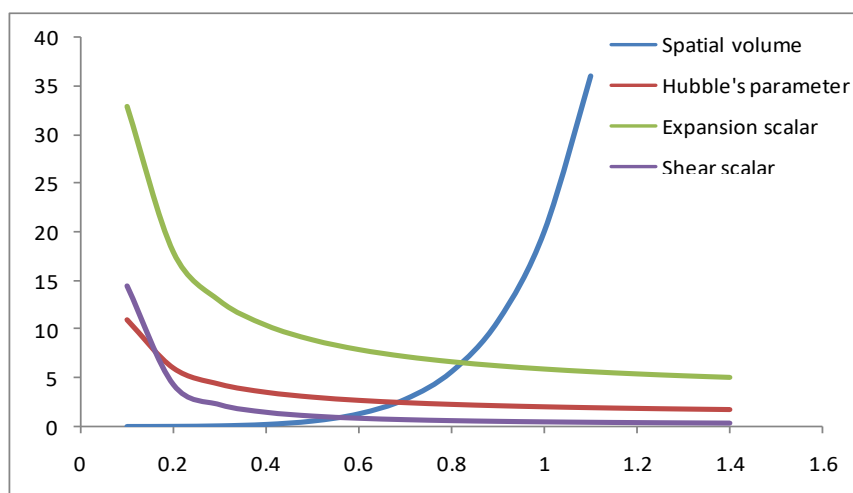
**Fig. 2:** Behavior of Spatial Volume, Hubble's Parameter, Expansion Scalar and Shear Scalar versus Time  $t$  for Some Particular Choice of Constants.



**Fig. 3:** Behavior of the Energy Density versus Time ( $t$ ), for Some Particular Choice of Constnt.



**Fig. 4:** Behavior of Deceleration Parameter versus Time  $t$ .



**Fig. 5:** Behavior of Spatial volume, Hubble's Parameter, Expansion Scalar and Shear Scalar versus Time  $t$  for Some Particular Choice of Constant.

## CONCLUSION

In both models, initially at  $t = 0$ , the spatial volume starts with big bang and expands with increase of time. The mean Hubble parameter and expansion scalar, both are infinitely large and gradually decrease with the expansion of the universe. Hence, the universe starts with infinite rate and then declines.

In model-I, the deceleration parameter shows a signature flipping i.e. universe starts with decelerating phase to recent accelerating phase, while model-II shows only accelerated expansion.

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