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Quark Contribution to the Magnetic Moments of Hypernuclei with Closed Core +Λ Configuration $\left({}^{5}_{\Lambda}He, {}^{13}_{\Lambda}\mathrm{C}, {}^{17}_{\Lambda}\mathrm{O}, {}^{41}_{\Lambda}\mathrm{Ca}\right)$

Madhulika Mehrotra*
Department of Physics, Isabella Thoburn College, University of Lucknow, Lucknow, Uttar Pradesh, India

Abstract

We have estimated the magnetic moments of these nuclei $\left({}^{5}_{\Lambda}He, {}^{13}_{\Lambda}\mathrm{C}, {}^{17}_{\Lambda}\mathrm{O}, {}^{41}_{\Lambda}\mathrm{Ca}\right)$ *in the Hybrid Quark Model. When the Λ-hyperon forms a six-quark bag with the core nucleons, the Schmidt values get modified. In the case of hypernuclei with closed core+ Λ configuration, the quark correction ranges between 0.1 to 2.1 %, the deviation being largest for* ${}^{41}_{\Lambda}Ca$ *. Earlier studies have shown that the comparison of predicted magnetic moments, with those of others calculations results very close to Schmidt value and has shown that correction arising due to various renormalization processes tend to cancel each other Thus any departure from the Schmidt values in the observed magnetic moments can be attributed to the quark effects.*

Keywords: *Hypernuclei, six-quark probability, Schmidt values, hybrid quark model (HQM)*

****Author for Correspondence*** E-mail: madhulikamehrotra@gmail.com

INTRODUCTION

In the conventional nuclear theory, the nuclear shell model works well for the calculation of nuclear magnetic dipole moments of nuclei. In the independent particle model approach magnetic moments of nuclei with configuration of closed core +1 nucleon are given by Schmidt values [1]. The deviation of these values from the experimental data has been a subject of continuous interest since long. In a somewhat similar approach Takeuchi and Shimizu (1986) [2] have estimated the effect of quark exchange currents on the magnetic moments of light hypernuclei and Gamow-Teller type β-decay matrix elements and have observed that these effects cause quenching in the free particle values.

Radhey Shyam et al. (1988) [3] have calculated the possibility of formation of six-quark cluster by overlapping of nucleons. The nucleons with the radius of 0.9 fm can partly overlap in the nucleus and form a cluster of six-quarks. The overlapping of nucleons and formation of six-quark bag can be the reason for deviations of the magnetic moments from the single particle values. This Hybrid Quark Model (HQM) approach was used by these authors showed that if magnetic moment of deuteron has an admixture of six-quark states with probability ranging between 3 to 6%, the experimental values can be explained for different types of nucleon-nucleon interaction.

A number of studies of the various characteristics of the nuclear ground state and of the nuclear dynamics have been made in the relativistic mean field models [4]. these models based on Dirac equation with strong scalar and vector potentials were originally designed for spherical systems and later extended to deformed ones [5]. In particular, attention has been focused on areas for which relativistic predictions differ significantly from those obtained in the traditional framework of nonrelativistic theory, for example the study of the nuclear currents and magnetic moments in particular. Both the relativistic and nonrelativistic models yield similar predictions for the isoscalar nuclear magnetic moments. In the relativistic mean field calculations, the result comes from a compensation of two effects, the enhancement of the valence particle current due to the reduction of the effective nucleon mass and the contribution of the additional current from polarized core nucleons [6], while the nonrelativistic (Schmidt) values are obtained directly in a single-particle approach. In case of hypernuclei, enhancement of the current caused by the valence hyperon is not entirely cancelled by the core corrections, since the mass of the hyperon and coupling constants are different from those of the nucleon. Moreover, the Λ single-particle current does not contribute to the hypernuclear magnetic moment (Λ is a neutral particle) and the Schmidt value is entirely due to the anomalous moment of the Λ-hyperon. But the non-vanishing current from the polarized core is the source of significant deviation from the Schmidt values. Cohen et al. (1987) [7] have studied the magnetic moments of ${}^{13}_{\Lambda}C, {}^{17}_{\Lambda}O, {}^{41}_{\Lambda}Ca, {}^{91}_{\Lambda}Zr, {}^{209}_{\Lambda}Pb$ in the mean field approximation to an extended σ-ω model and has estimated the values of magnetic moments in a spherical core, where core response to the added particle is calculated in the random phase approximation. Mares and Zofka (1990) [8] have also estimated the magnetic moments of the same set of hypernuclei in the mean field model with a deformed core. Both these studies predict similar and rather sizable corrections to the Schmidt values ranging from 8% to 12%. As explained above in the case of Λ-hyperon orbiting a nucleus, the nucleon valence particle current is not totally cancelled, as the coupling of the valence particle Λ to the vector meson is different from that for the nucleons. However, these calculations omitted the very important tensor coupling of the vector field (ω) to the Λ-hyperon [9]. This range in the Λ-coupling induces a modification in the sector of baryon-hyperon current and has implications on the structure of the backflow current for finite nuclei.

In the later studies Gottane et al. (1991) [10], Cohen and Noble (1992) [11] have calculated the magnetic moments of several hypernuclei within the framework of σ-ω model and have shown that the inclusion of tenser term to describe the strong σ-ω coupling restores the magnetic moments to values very close to the Schmidt limit. Dover et al. (1995) [12] have shown that $\Lambda-\Sigma$ mixing due to the strong $\Lambda N \leftrightarrow \Sigma N$ interaction can also produce deviation of hypernuclear magnetic moments from the Schmidt values. These authors have also observed that $\Sigma-\Lambda$ mixing becomes more effective when there is no change in the nuclear core states and deviation from Schmidt values are large for p-shell hypernuclei compared to that of the s-shell hypernuclei.

In the present work, we have calculated the magnetic moments of hypernuclei with configuration closed core + Λ-hyperon. It is well known that the baryonic properties change inside a nuclear medium, thus the magnetic moment of Λ-hyperon well inside a six-quark bag of ΛN would be different from that of the free hyperon. This can be a source of deviation of the magnetic moment values from the Schmidt limits.

CALCULATION

In the hybrid quark model nucleons exhibit quark degrees of freedom and form a six-quark bag, when the separation between two nucleons is less than a certain critical radius r_0. When the nucleon resides in a six-quark bag, its effective magnetic moment is increased. This is natural in a bag model, where the magnetic moment of a quark is proportional to the radius of the bag [13]. In the same bag model when the numbers of quarks are doubled from three to six, the bag radius is increased by a factor of about 4/3. This factor results from the bag model theorem, in which the mass of the bag is proportional to its volume. The mass of the six-quark bag is about twice that of a three-quark bag plus some 300 MeV, which accounts for the repulsive core. Therefore, in the quark picture the effective magnetic moment of the nucleon is larger by about a factor of 4/3 [14]. Thus, in the hybrid quark model framework the Schmidt expression for hypernuclei with closed core + Λ configuration, the magnetic moments in the extreme single-particle shell model, where Λ-hyperon is in the *j* state are given as,

$$\mu_{Sch} = -\left(\frac{j}{j+1}\right)\mu_{\Lambda} \quad \Big| j = l - \frac{1}{2} \tag{1}$$

And

$$\mu_{Sch} = \mu_{\Lambda} \quad \Big| j = l + \frac{1}{2} \tag{2}$$

When the Λ-hyperon forms a six-quark bag with the core nucleons with a probability $P^{6q}_{\Lambda N}(r_0)$, equations (1) and (2) gets modified to,

$$\mu_{Cal} = -\left(\frac{j}{j+1}\right)\left[1-P^{6q}_{\Lambda N}(r_0)\mu_\Lambda + P^{6q}_{\Lambda N}(r_0)\mu'_\Lambda\right] \Big| j = l - \tfrac{1}{2} \qquad (3)$$

And

$$\mu_{Cal} = \left[\left(1-P^{6q}_{\Lambda N}\left(r_0\right)\right)\mu_\Lambda + P^{6q}_{\Lambda N}\left(r_0\right)\mu'_\Lambda\right] \big| j = l + \tfrac{1}{2} \qquad (4)$$

We have also estimated the magnetic moments of hypernuclei with closed core + Λ configuration $\left({}^{5}_{\Lambda}He, {}^{13}_{\Lambda}\mathrm{C}, {}^{17}_{\Lambda}\mathrm{O}, {}^{41}_{\Lambda}\mathrm{Ca}\right)$ using equations (3) and (4). Where $P^{6q}_{\Lambda N}(r_0)$ is the average probability for the formation of six-quark bag with the overlap of hyperon with various core nucleons. These probabilities have been calculated both in the Moshinsky and Slater [15] method for different sets of oscillator length parameters (Mujib et al., 1979 [16] and Motoba et al., 1994 [17] for ${}^{5}_{\Lambda}He$ ${}^{13}_{\Lambda}C$ Shlomo, 1972 [18] and Modarres, 1994 [19] for ${}^{17}_{\Lambda}O$ ${}^{41}_{\Lambda}Ca$) shown in Table 1. The predicted magnetic moments are given in Table 2 (Moshinsky method) and Table 3 (Slater method) both for $\mu'_\Lambda = \tfrac{4}{3}\mu_\Lambda$ and $\mu'_\Lambda = 1.08\mu_\Lambda$. The results obtained in the present work are compared with those of other authors in Table 4.

***Table 1:** The Values of Oscillator Length Parameters for Nucleon $\left(\nu_N\right)$ and Hyperon $\left(\nu_\Lambda\right)$ Taken by Different Authors. The Values of $\left(\nu_N\right)$ Marked with* are Obtained by using Relation $\nu_N = \left(m_N / m_\Lambda\right)\nu_\Lambda$.*

Hypernuclei	Set I		Set II	
	$\nu_N\left(fm^{-2}\right)$	$\nu_\Lambda\left(fm^{-2}\right)$	$\nu_N\left(fm^{-2}\right)$	$\nu_\Lambda\left(fm^{-2}\right)$
${}^{5}_{\Lambda}He^{a}$	0.521 0.521	0.619* 0.336	0.532 0.521	0.632* 0.457
${}^{13}_{\Lambda}C^{a}$	0.359 0.359	0.426* 0.214	0.370 0.359	0.440* 0.287
${}^{17}_{\Lambda}O^{b}$	0.338	0.402*	0.397	0.472*
${}^{41}_{\Lambda}Ca^{b}$	0.25	0.297*	0.26	0.309*

a, Mujib et al. (1979) [16] and Motoba et al. (1994) [17]; b, Shlomo (1972) [18] and Madarres (1994) [19].

***Table 2:** The Magnetic Moments of Hypernuclei with Closed Core + Λ Configuration. Six-quark Probabilities are Calculated by Moshinsky method with $r_0 = 1\,fm$.. The Effective Magnetic Moment of a Hyperon μ'_Λ Inside a Six-quark Bag is Taken to be $\mu'_\Lambda = \tfrac{4}{3}\mu_\Lambda = -0.817 n.m.$. The Values Marked with* are Obtained with $\mu'_\Lambda = 1.08\mu_\Lambda = -0.662 n.m.$*

Hypernuclei	J^{π}	Set I		Set II	
		$P^{6q}_{\Lambda N}\left(r_0\right)$	μ (n.m.)	$P^{6q}_{\Lambda N}\left(r_0\right)$	μ (n.m.)
${}^{5}_{\Lambda}He^{a}$	$\frac{1}{2}^{+}$	0.0439	-0.621 -0.614*	0.0451	-0.622 -0.615*
${}^{13}_{\Lambda}C^{a}$	$\frac{1}{2}^{+}$	0.304	-0.619 -0.614*	0.0317	-0.619 -0.615*
${}^{17}_{\Lambda}O^{b}$	$\frac{1}{2}^{+}$	0.0497	-0.623 -0.615*	0.0624	-0.625 -0.616*
${}^{41}_{\Lambda}Ca^{b}$	$\frac{1}{2}^{+}$	0.0701	-0.627 -0.616*	0.0725	-0.628 -0.617*

For ${}^{5}_{\Lambda}He$ and ${}^{13}_{\Lambda}C$ results are presented for oscillator length parameters obtained from 2nd and 4th rows of Table 1.

Table 3: *The Magnetic Moments of Hypernuclei with Closed Core* $+\ \Lambda$ *Configuration. Six-quark Probabilities are Calculated by Slater Method with* $r_0 = 1\,fm.$*. The Effective Magnetic Moment of a Hyperon* μ'_Λ *Inside a Six-quark Bag is Taken to be* $\mu'_\Lambda = {}^4\!/_3\,\mu_\Lambda = -0.817 n.m.$ *The Values Marked with* are Obtained with* $\mu'_\Lambda = 1.08\mu_\Lambda = -0.662 n.m.$

Hypernuclei	J^π	**Set I**		**Set II**	
		$P^{6q}_{\Lambda N}(r_0)$	μ **(n.m.)**	$P^{6q}_{\Lambda N}(r_0)$	μ **(n.m.)**
${}^{5}_{\Lambda}He^{a}$	$\frac{1}{2}^{+}$	0.0559	-0.624 -0.616*	0.0715	-0.628 -0.617*
${}^{13}_{\Lambda}C^{a}$	$\frac{1}{2}^{+}$	0.0513	-0.624 -0.616*	0.0630	-0.626 -0.616*
${}^{17}_{\Lambda}O^{b}$	$\frac{1}{2}^{+}$	0.0720	-0.628 -0.617*	0.0903	-0.632 -0.617*
${}^{41}_{\Lambda}Ca^{b}$	$\frac{1}{2}^{+}$	0.0990	-0.633 -0.618*	0.1049	-0.634 -0.618^{+}

All results are presented for oscillator length parameters marked with* in 1st, 3rd, 5th and 6th rows of Table 1.

Table 4: *The Comparison of Predicted Magnetic Moments of Hypernuclei with Closed Core* $+\ \Lambda$ *Hypernuclei, with Those of Others Calculations.*

Reference	μ **(n.m.)**			
	${}^{5}_{\Lambda}He$	${}^{13}_{\Lambda}C$	${}^{17}_{\Lambda}O$	${}^{41}_{\Lambda}Ca$
Present Work				
MM1	-0.621	-0.619	0.623	-0.627
MM2	-0.614	-0.614	0.615	-0.616
SM1	-0.624	-0.624	0.628	-0.633
SM2	-0.616	-0.616	0.617	-0.618
Others Work				
Cohen[a]	--	-0.650	-0.648	-0.665
Mores[b]	--	-0.649	-0.643	-0.656
Gattone[c]	--	-0.611	-0.611	-0.611
Schmidt	--	-0.613	-0.613	-0.613

Note: In the present work result are tabulated for parameters of set I with cut off radius $r_0 = 1\,fm.$

MM1: Moshinsky method ($\mu'_\Lambda = {}^4\!/_3\,\mu_\Lambda$); MM2: Moshinsky method ($\mu'_\Lambda = 1.08\mu_\Lambda$);

SM1: Slater method ($\mu'_\Lambda = {}^4\!/_3\,\mu_\Lambda$); SM2: Slater method ($\mu'_\Lambda = 1.08\mu_\Lambda$).

a, Cohen et al. (1987). [7]; b, Mares et al. (1990) [9]; c, Gattone et al. (1991) [11].

We have also plotted a comparative graph (Figure 1) which shows the magnetic moments of ${}^{5}_{\Lambda}He$, ${}^{13}_{\Lambda}C$, ${}^{17}_{\Lambda}O$ and ${}^{41}_{\Lambda}Ca$, obtained in the present work, others calculations and the Schmidt value.

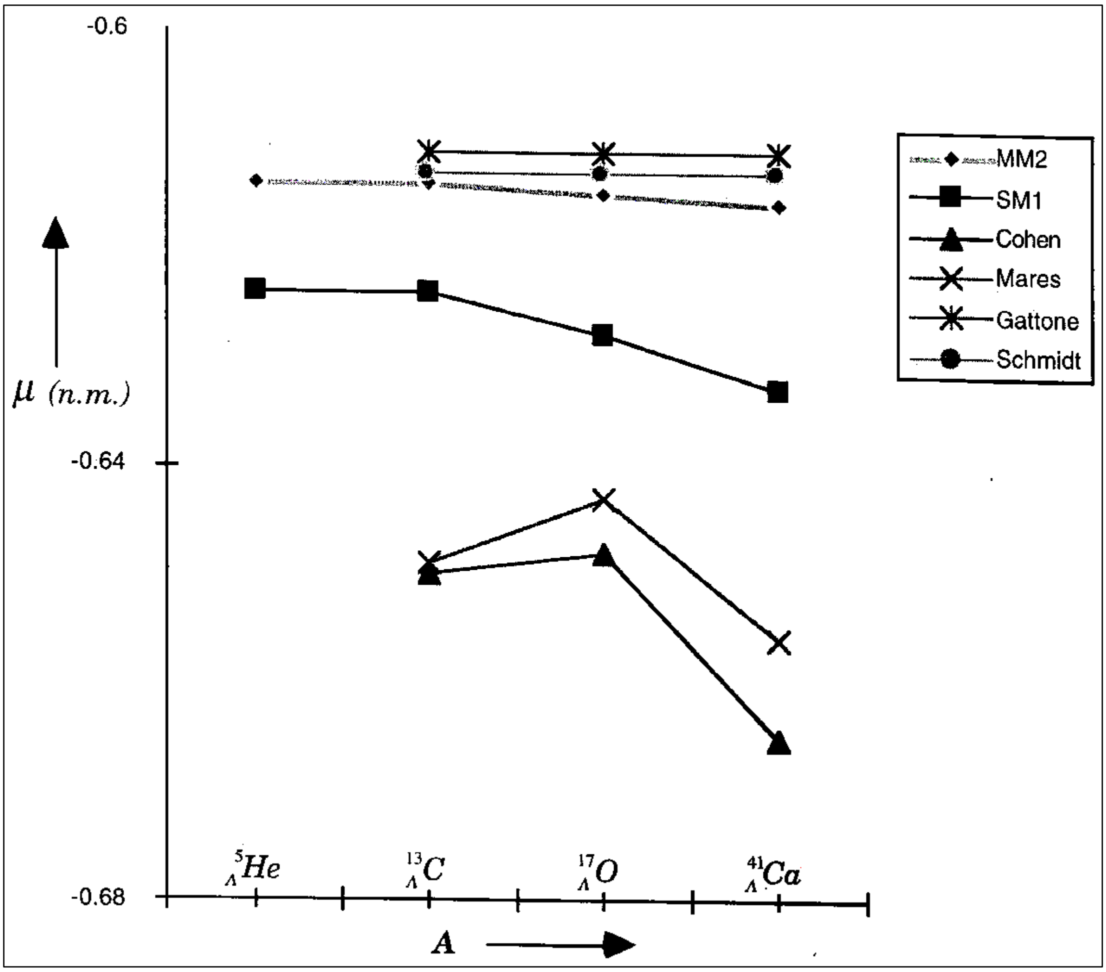

***Fig. 1:** Plot of Magnetic Moments (μ) of Hypernuclei with Closed Core $+ \Lambda$ Configuration, with Mass Number (A).*

CONCLUSION

It is observed that the magnetic moments of hypernuclei change by ~0.1 to 2.1% and the deviation is largest for $^{41}_{\Lambda}Ca$. Cohen et al. (1987) [7] have obtained the magnetic moments in the self-consistent calculations by evaluating the effect of the linear core response to the hyperon in the local density approximation. Mares et al. (1990) [8] have made self-consistent calculations of the entire core + Λ system and incorporated this effect automatically. Both these calculations have obtained remarkable deviation from the Schmidt values. Gattone et al. (1991) [10] have estimated the magnetic moments in the relativistic mean-field model including a tenser term to describe the $\sigma - \omega$ coupling and have obtained results very close to standard Schmidt value even in the presence of strong renormalization current. The comparison of predicted magnetic moments, with those of others calculations as shown in Table 4 and in Figure 1, we can say that Gattone et al. (1991) [10] has obtained results very close to Schmidt value and has shown that various renormalization processes tend to cancel each other. Thus again, any departure of the experimental values from the Schmidt values can be attributed to the quark effects.

The experimental data [20] on the measurement of magnetic moment of hypernuclei is still awaited. In hypernuclei, hyperon carries non-zero strangeness, which distinguishes them from nucleons. Thus they are excluded from orbital occupied by nucleons and can penetrate dense nuclear matter inaccessible to other hadronic probes. Hypernuclei can be used to explore the role of hyperons in hadronic forces and to measure changes in electromagnetic properties of hyperons by the nuclear environment.

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