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Magneto-thermodynamic Stress Analysis of an Orthotropic Solid Cylinder by Fractional Order Theory Application

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Abstract

This study presents the Magneto-thermodynamic Response of Orthotropic Solid Cylinder in context of fractional order theory. The Laplace transform and finite Marchi–Zgrablich transform techniques have been used to analyze the thermal behavior of solid cylinder. The Magneto-thermodynamic stresses and perturbation of magnetic field vector in finite orthotropic cylinder are obtained in the transformed domain under sudden temperature change to a constant temperature and permeated by uniform primary magnetic field. Numerical computed results of stresses and perturbation of magnetic field vector are depicted graphically for Caputo type time fractional equation of order α along radial direction. Some particular cases are also discussed in the context of the problem.

Keywords: *Magneto-thermodynamic, Laplace transform, Marchi–Zgrablich transform, stresses, Solid Cylinder*

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INTRODUCTION

Magneto-thermodynamic theory of thermoelasticity has aroused much interest and it has wide range of applications in many mathematical and engineering modeling such as in design of nuclear reactor, geothermal engineering, high accelerators energy particle, etc. Biot [1] initially introduced the theory of coupled thermoelasticity to overcome the first shortcoming. The governing equations in this coupled theory involved eliminating the first paradox of the classical theory. After this many generalizations to the coupled theory were introduced. Hetnarski and Ignaczak [2] examined several important analytical results for the coupled theory. Hetnarski and Eslami [3] introduced a unified generalized thermoelasticity theory and also successfully presented the mathematical and mechanical background of thermodynamics, classical thermoelasticity advanced theory and applications, generalized thermoelasticity. Preziosi [4] and Cattaneo [5] stated the Fourier law for a finite propagation speed.

Dhaliwal [6] dealt with the generalized elastic theory applied to the problems of magnetothermoelastic waves produced by thermal shocks in an infinite elastic solid with cylindrical cavity. Ezzat [7] found the distribution of thermal stresses and temperature in perfectly conducting half spaces when suddenly heated in absence of magnetic field. In Sherief [12], the Laplace transform technique was used to find the distribution of thermal stresses and temperature in a thermoelastic, electrically conducting half spaces under sudden thermal shock and permeated by magnetic field.

Sharma [14] evaluated the distribution of temperature, deformation and magnetic field in a homogeneous isotropic, thermally and perfectly electrically conducting, elastic half space in context of Green-Lindsey theory of thermoelasticity. Chandrasekharaiah [15] studied the propagation of magneto-thermo-elastic disturbances with thermal relaxation in a perfectly conducting unbounded solid, due to heat sources distributed over a plane parallel to the applied magnetic field. Choudhuri [16] investigated the propagation of magneto-thermoelastic disturbances produced by a thermal shock in a finitely conducting elastic half-space in contact with a vacuum. Ezzat [17] established the model of the two-dimensional equations of generalized magneto-thermoelasticity with two relaxation times in a perfectly conducting medium [18].

Povstenko [19] proposed A quasi-static uncoupled theory of thermoelasticity based on the heat conduction equation with a time-fractional derivative of order α. Mondal [20] constructed a new theory of two-temperature generalized thermoelasticity in the context of a new consideration of dual-phase-lag heat conduction with fractional orders.

Kalkal [21] studied the effects of fractional order parameter, magnetic field, viscosity and diffusion on the thermoelastic interactions in an infinite body whose surface suffers a mechanical load.

NOTATION AND GOVERNING EQUATIONS

The basic relationship for the problem defined above can be summarized as follows:

1. *The Caputo type fractional derivative for nonlocal heat conduction is defined by* [15]

$$\frac{\partial^\alpha f(t)}{\partial t^\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{d^n f(\tau)}{d\tau^n} d\tau, \qquad n-1<\alpha<n \tag{1}$$

To find Laplace transforms of the Caputo derivative it needs to know the initial values of the function $f(t)$ and its integer derivatives of the order $P = 0,1,2,\ldots,n-1$

$$L\left\{\frac{\partial^\alpha f(t)}{\partial t^\alpha}\right\} = s^\alpha f^*(s) - \sum_{P=0}^{P=n-1} f^{(P)}(0^+) s^{\alpha-1-P}, \qquad n-1<\alpha<n \tag{2}$$

2*. The governing electrodynamics Maxwell's equation of fractional order α for perfectly conducting body is given as:*

$$\left.\begin{aligned} &\bar{J} = curl\, \bar{h} \\ &-\mu \frac{\partial^\alpha \bar{h}}{\partial t^\alpha} = curl\, \bar{e} \\ &div\, \bar{h} = 0 \\ &\bar{e} = -\mu\left(\frac{\partial^\alpha \bar{U}}{\partial t^\alpha} \times \bar{H}\right) \end{aligned}\right\} \tag{3}$$

where, J is current density, h is perturbation of magnetic field vector $(0, 0, h_Z)$, e is perturbation of electric field vector, H is magnetic intensity vector $(0, 0, H_z)$ and U is the displacement vector. Here the magnetic permeability, μ, of the orthotropic solid cylinder is assumed to be equals the magnetic permeability of the medium around it.

3. Stress-strain relationship*:* The generalized Hooke's law for an orthotropic thermoelastic cylinder from Lekhnitskii [10] can be written as

$$\varepsilon_r = \frac{1}{E_r}\sigma_r - \frac{\nu_{\theta r}}{E_\theta}\sigma_\theta - \frac{\nu_{zr}}{E_z}\sigma_z + \alpha_r T \tag{4}$$

$$\varepsilon_\theta = -\frac{\nu_{\theta r}}{E_\theta}\sigma_r + \frac{1}{E_\theta}\sigma_\theta - \frac{\nu_{z\theta}}{E_z}\sigma_z + \alpha_\theta T \tag{5}$$

$$\varepsilon_z = -\frac{\nu_{zr}}{E_z}\sigma_r - \frac{\nu_{z\theta}}{E_z}\sigma_\theta + \frac{1}{E_z}\sigma_\theta + \alpha_z T \tag{6}$$

Here, $\varepsilon_i (i = r, \theta, z)$ denotes the strains components also σ_r, σ_θ are radial and circumferential stress. E_i, $\nu_{i,j}$ α_i $(i, j = r, \theta, z)$ are the Young's modulus, Poison's ratio and coefficient of thermal expansion in radial, circumferential and axial directions respectively.

4. Stress-Displacement relationship*:* Considering a generalized plane strain problem, $\varepsilon_z = 0$ and solving equation (4) to (6) we get

$$\sigma_r = s_1\left[\left(\frac{1}{E_\theta} - \frac{\nu_{z\theta}^2}{E_z}\right)\frac{\partial U_r}{\partial r} + \left(\frac{\nu_{\theta r}}{E_\theta} - \frac{\nu_{z\theta}\nu_{zr}}{E_\theta}\right)\frac{U_r}{r} - s_2 T\right] \tag{7}$$

$$\sigma_\theta = s_1\left[\left(\frac{\nu_{\theta r}}{E_\theta} - \frac{\nu_{z\theta}\nu_{zr}}{E_\theta}\right)\frac{\partial U_r}{\partial r} + \left(\frac{1}{E_r} - \frac{\nu_{zr}^2}{E_z}\right)\frac{U_r}{r} - s_3 T\right] \tag{8}$$

Where, constants s_1, s_2, s_3 are as shown

$$s_1 = \frac{1}{\left(\frac{1}{E_r} - \frac{\nu_{zr}^2}{E_z}\right)\left(\frac{1}{E_\theta} - \frac{\nu_{z\theta}^2}{E_z}\right) - \left(\frac{\nu_{\theta r}}{E_\theta} - \frac{\nu_{z\theta}\nu_{zr}}{E_\theta}\right)^2},$$

$$s_2 = \left(\frac{1}{E_\theta} - \frac{\nu_{z\theta}^2}{E_z}\right)(\alpha_r + \nu_{zr}\alpha_z) + \left(\frac{\nu_{\theta r}}{E_\theta} - \frac{\nu_{z\theta}\nu_{zr}}{E_\theta}\right)b_2 = \alpha_\theta + \nu_{z\theta}\alpha_z,$$

$$s_3 = \left(\frac{\nu_{\theta r}}{E_\theta} - \frac{\nu_{z\theta}\nu_{zr}}{E_\theta}\right)(\alpha_r + \nu_{zr}\alpha_z) + \left(\frac{1}{E_r} - \frac{\nu_{zr}^2}{E_z}\right)(b_2 = \alpha_\theta + \nu_{z\theta}\alpha_z)$$

FORMULATION OF THE PROBLEM

Consider a long orthotropic solid cylinder of radius a placed in an axial magnetic field $H(0, 0, H_z)$. If electromagnetic pulse or γ rays pulse radiant energy is incident on a solid cylinder, then it undergoes rapid change in temperature $T(r,t)$. This then leads to interactions between the deformations of the cylinder and the perturbation of the magnetic field vector in orthotropic cylinder.

Applying the initial magnetic field $H(0,0, H_z)$ to equation (3) in (r,θ,z) i.e. cylindrical polar coordinate system, we get

$$\left.\begin{aligned} \bar{U} &= (\bar{U}_r, 0, 0) \\ \bar{H} &= (0, 0, \bar{H}_z) \\ \bar{e} &= \left(0, -\bar{H}_z\frac{\partial^\alpha \bar{U}_r}{\partial t^\alpha}, 0\right) \end{aligned}\right\} \tag{9}$$

$$\bar{h}_z = curl\,(\bar{U} \times \bar{H}) = \left[0, 0, -\bar{H}_z\left(\frac{1}{r}\frac{\partial}{\partial r}(r, U_r)\right)\right]$$

$$= \left[0, 0, -\bar{H}_z\left(\frac{\partial U_r}{\partial r} + \frac{U_r}{r}\right)\right] \tag{10}$$

$$h = (0, 0, \bar{h}_z), \quad J = \left(0, \frac{\partial \bar{h}_z}{\partial r}, 0\right) \tag{11}$$

$$\bar{h}_z = -\bar{H}_z\left(\frac{\partial U_r}{\partial r} + \frac{U_r}{r}\right) \tag{12}$$

U_r is the radial displacement component.

From equations (3), (9), (10), (11), (12) and **Fung [8]** the magneto elastic dynamic equation in time fractional order context of orthotropic cylinder becomes

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}(\sigma_r - \sigma_\theta) + f_r = \rho \frac{\partial^{1+\alpha} U_r}{\partial t^{1+\alpha}} \qquad 0 < \alpha < 1 \tag{13}$$

Where, f_r is the Lorentz force [**9**] which is given by

$$f_r = \mu(\bar{J} \times \bar{H}) = \mu H_z^2 \frac{\partial}{\partial r}\left(\frac{\partial U_r}{\partial r} + \frac{U_r}{r}\right) \tag{14}$$

Substituting equations (7) and (8) into equation (13), the basic displacement equation of magnetothermoelastic motion in an orthotropic cylinder may be expressed as in Wang et al [11].

$$\frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r}\frac{\partial U_r}{\partial r} - N^2 \frac{1}{r^2} U_r = \frac{1}{V^2}\frac{\partial^{1+\alpha} U_r}{\partial t^{1+\alpha}} + k_1 \frac{\partial T}{\partial r} + k_2 \frac{T}{r}; \tag{15}$$

Where, $0 \le r \le a$, $t \ge 0$

$$N^2 = \frac{s_1\left(\frac{1}{E_r} - \frac{\nu_{zr}^2}{E_z}\right) + \mu H_z^2}{s_1\left(\frac{1}{E_\theta} - \frac{\nu_{z\theta}^2}{E_z}\right) + \mu H_z^2}, v^2 = \frac{s_1\left(\frac{1}{E_\theta} - \frac{\nu_{z\theta}^2}{E_z}\right) + \mu H_z^2}{\rho}, k_1 = \frac{s_1 s_2}{s_1\left(\frac{1}{E_\theta} - \frac{\nu_{z\theta}^2}{E_z}\right) + \mu H_z^2}$$

$$k_2 = \frac{s_1(s_2 - s_3)}{S_1\left(\frac{1}{E_\theta} - \frac{\nu_{z\theta}^2}{E_z}\right) - \mu H_z^2}$$

Omitting the Maxwell tensor on the surface of orthotropic cylinder, the corresponding boundary conditions are

$U_r = 0$ at $r = 0$

$$\sigma_r(0,t)|_{r=a} = \left[s_1\left(\left(\frac{1}{E_\theta} - \frac{\nu_{z\theta}^2}{E_z}\right)\frac{\partial U_r}{\partial r} + \left(\frac{\nu_{\theta r}}{E_\theta} - \frac{\nu_{z\theta}\nu_{zr}}{E_\theta}\right)\frac{U_r}{r} - s_2 T\right)\right]_{r=a} = 0 \tag{16}$$

The initial conditions are

$$U_r(r,t) = 0 \qquad \text{at } t = 0 \tag{17}$$

$$\frac{\partial U_r}{\partial t}(r,t) = 0 \qquad \text{at } t = 0 \tag{18}$$

SOLUTION OF THE PROBLEM

Let us assume that the general solution to equation (15) may be expressed in the form

$$U(r,t) = V(r,t) + W(r,t) \tag{19}$$

Where, $V(r,t)$ is the static part of solution to equation (15) and $W(r,t)$ is the dynamic part of solution to equation (15). The static part V satisfies the solution with inhomogeneous boundary conditions while the dynamic part W satisfies the solution with homogenous boundary conditions. For the static part $V(r,t)$, the governing equations and boundary conditions become

$$\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}-N^2\frac{1}{r^2}V=k_1\frac{\partial T}{\partial r}+k_2\frac{T}{r} \tag{20}$$

$$V=0 \text{ at } r=0 \tag{21}$$

$$\left.\left(\frac{1}{E_\theta}-\frac{\nu_{z\theta}^2}{E_z}\right)\frac{\partial V}{\partial r}+\left(\frac{\nu_{\theta r}}{E_\theta}-\frac{\nu_{z\theta}\nu_{zr}}{E_\theta}\right)\frac{V}{r}\right]_{r=a}=S_2T(a)\delta(t) \tag{22}$$

Equation (20) simplifies to

$$\frac{\partial}{\partial r}\left[r^{-(2N-1)}\frac{\partial}{\partial r}(r^N V)\right]=r^{-N+1}\left(k_1\frac{\partial T}{\partial r}+k_2\frac{T}{r}\right) \tag{23}$$

From equation (23) the solution of equation (20) which satisfies the boundary conditions (21) is written as

$$V(r,t)=C_1r^N+r^{-N}\int_0^r r^{2N-1}[\int_0^r r^{-N+1}\left(k_1\frac{\partial T}{\partial r}+k_2\frac{T}{r}\right)dr]\,dr \tag{24}$$

Where,

$$N=\sqrt{\frac{S_1\left(\frac{1}{E_r}-\frac{\nu_{zr}^2}{E_z}\right)+\mu H_z^2}{S_1\left(\frac{1}{E_\theta}-\frac{\nu_{z\theta}^2}{E_z}\right)+\mu H_z^2}}>0 \tag{25}$$

The unknown constant C_1 in equation (24) can be easily determined from boundary condition (22). The dynamic part of the solution $W(r,t)$ should satisfy the inhomogeneous equation (26) and corresponding homogenous boundary conditions (27) and (28) and initial condition (29).

$$\frac{\partial^2 W}{\partial r^2}+\frac{1}{r}\frac{\partial W}{\partial r}-\frac{N^2}{r^2}W=\frac{1}{\psi^2}\left[\frac{\partial^{1+\alpha}W}{\partial t^{1+\alpha}}+\frac{\partial^{1+\alpha}V}{\partial t^{1+\alpha}}\right] \tag{26}$$

$$W=0 \quad \text{at } r=0 \tag{27}$$

$$\left[\left(\frac{1}{E_\theta}-\frac{\nu_{z\theta}^2}{E_z}\right)\frac{\partial W}{\partial r}+\left(\frac{\nu_{\theta r}}{E_\theta}-\frac{\nu_{z\theta}\nu_{zr}}{E_\theta}\right)\frac{W}{r}\right]_{r=a}=0 \tag{28}$$

$$W=-V=0 \quad at \quad t=0,\ \frac{\partial^\alpha W}{\partial t^\alpha}=-\frac{\partial^\alpha V}{\partial t^\alpha}=0 \quad at \quad t=0 \tag{29}$$

Where, V is the static solution as on equation (24), and ψ is Magneto thermoelastic wave speed.

Applying finite Marchi–Zgrablich (defined in Appendix) and Laplace transform and their inversions (defined in Equation (1) and (2)) to equation (26) and making use of the transformed boundary and initial conditions (27)–(29), one obtains temperature distribution function expressed as follows,

$$W(r,t)=\sum_{n=1}^{\infty}\left[\frac{S_p(\alpha_1\ \beta_1\ \mu_n r)}{\int_0^a r[S_p(\alpha,\beta,\mu_n r)]^2 dr}\right]\left(-\bar{V}(n,t)+\mu_n\,\psi\,E_\alpha\left(-\psi\left(\mu_n^2\right)(t^\alpha-t'^\alpha)\right)+\bar{V}_0\cos(\mu_n\psi\,t)+\frac{\bar{W}_0}{\mu_n\psi}\sin(\mu_n\psi\,t)\right) \quad (30)$$

Where, $\bar{V}(n,t)$ is finite Marchi-Zgrablich integral transform of $V(r,t)$ with respect to the Kernel function $S_p(\alpha,\beta,\mu_n\,r)$ and weight function r. Here the Kernel $S_p(\alpha,\beta,\mu_n\,r)$ can be defined as

$$S_p(\alpha,\beta,\mu_n\,r)=J_p(\mu_n\,r)$$
$$[Y_p(\alpha,\mu_n 0)+Y_p(\beta,\mu_n a)]-Y_p(\mu_n r)\,[J(\alpha,\mu_n 0)+J_p(\beta,\mu_n a)] \quad (31)$$

Where, $J_p(k_i,\mu\xi)=J_p(\mu\xi)+k_i\,\mu\,J_p^1(\mu\xi)$

$$Y_p(k_i\ \mu\xi)=Y_p(\mu\xi)+k_i\,\mu\,Y_p^1(\mu\xi)\ \text{for}\ i=1,2,3,\ldots \quad (32)$$

$J_p(\mu r)$ and $Y_p(\mu r)$ are Bessel's function of first and second kind respectively.

By substituting equations (24) and (30) in equation (19) the general solution for the basic governing equation (15) becomes

$$U(r,t)=C_1 r^N+r^{-N}\int_0^r r^{2N-1}\left[\int_0^r r^{-N+1}\left(k_1\frac{\partial T}{\partial r}+k_2\frac{T}{r}\right)dr\right]dr$$
$$+\sum_{n=1}^{\infty}\left[\frac{S_p(\alpha_1\ \beta_1\ \mu_n r)}{\int_0^a r[S_p(\alpha,\beta,\mu_n r)]^2 dr}\right]\left(-\bar{V}(n,t)+\mu_n\,\psi\,E_\alpha\left(-\psi\left(\mu_n^2\right)(t^\alpha-t'^\alpha)\right)+\bar{V}_0\cos(\mu_n\psi\,t)+\frac{\bar{W}_0}{\mu_n\psi}\sin(\mu_n\psi\,t)\right) \quad (33)$$

By substituting equation (33) in equations (7) and (8), we can obtain the corresponding Magneto thermodynamic stress. Similarly the perturbation response of the magnetic field vector can be obtained by substituting equation (33) in equation (12).

NUMERICAL ANALYSIS

The symmetric magnetothermoelastic problem of an orthotropic solid cylinder placed in uniform axial magnetic field and subjected to thermal shocks due to any source is considered. Here for the orthotropic solid cylinder the various material constants taken are:

$$E_Z=150\,GP_a\ ,\ E_r=7.5\,GP_a\ ,\ E_\theta=20GP_a$$
$$\nu_{zr}=0.25,\ \nu_{z\theta}=0.3,\ \nu_{\theta r}=0.2$$
$$\alpha_r=1\times10^{-4}\frac{1}{{}^0C},\ \alpha_\theta=1\times10^{-5}\frac{1}{C^0},\ \alpha_z=1\times10^{-6}\frac{1}{{}^0C}$$

Magnetic interference wave speed $\psi=1000$ m/s. For the sake of convenience, we choose the radius of a thin circular plate to be *a= 1 m*.

Figures 1–3 show the response histories for radial stress, circumferential stress, and magnetic field vector perturbations for fractional-order parameter $\alpha=0.5$, 1 ,1.5 *and* 2 depicting weak, normal, and strong conductivity and fixed time $t=0.6$. Also solid cylinder subjected to thermal shocks and placed in uniform axial magnetic field respectively. For numerical calculation MATLAB 2013 has been carried out in a programming environment and here the Mittag–Leffler functions were evaluated following Podlubny [22].

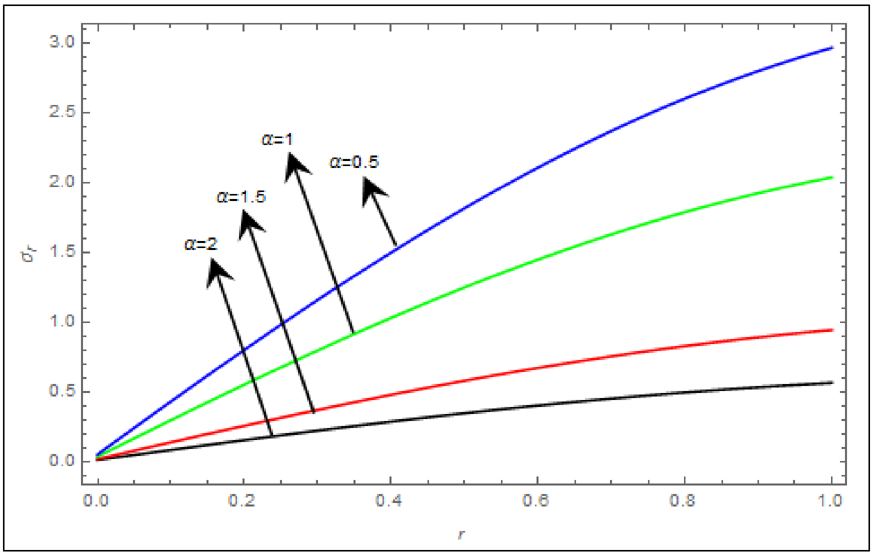

Fig. 1: *Radial Stress Distribution.*

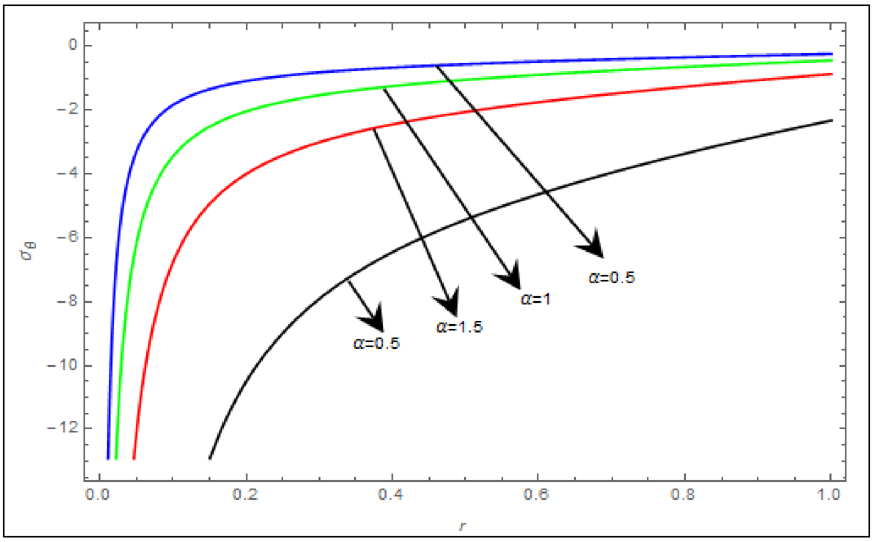

Fig. 2: *Circumferential Stress Distribution.*

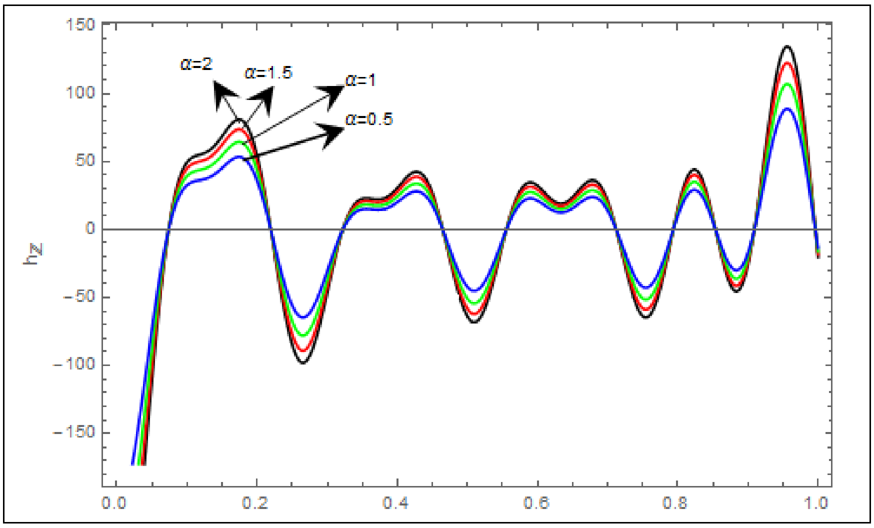

Fig. 3: *Magnetic Field Vector Perturbations.*

Figure 1 shows the radial stresses distribution for different value of radius. It is clear that radial stress have zero magnitude initially. For different $\alpha = 0.5\,, 1\,, 1.5\ and\ 2$ radial stress value goes on increasing as moving radially outward.

Figure 2 shows response curves for circumferential stress along the radial direction. For different $\alpha = 0.5\,, 1\,, 1.5\ and\ 2$ circumferential stress value goes on increasing in the range $0 \le r \le 0.2$ and then start converges towards radially outward direction. A stress–focusing effect also observed near the center in a conducting orthotropic solid cylinder placed in axial magnetic field.

Figure 3 represents the response curves for perturbation of magnetic field vector in orthotropic solid cylinder for different value of $\alpha = 0.5\,, 1\,, 1.5\ and\ 2$. A complex distribution of perturbation responses appear as going radially outward.

CONCLUSION

The solutions for displacement, stress components and magnetic field vector perturbations were found by using the Laplace transform and finite Marchi–Zgrablich transforms for conducting orthotropic solid cylinder placed in axial magnetic field. The effect of fractional heat conduction with their thermoelasticity by quasi-static approach has been discussed in this work. Stress distributions and magnetic field vector perturbations plotted for different $\alpha = 0.5\,, 1\,, 1.5\ and\ 2$. By using this we can design various magnetoelastic elements for specific engineering requirements where fractional differential operator describes memory effects, and space fractional differential operator deals with the long-range interactions. The knowledge can also be used for proper selection of materials for in designing and manufacturing field; and to control the magnetothermoelastic stresses and magnetic field vector perturbations.

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APPENDIX

A Short Note on Finite Marchi-Zgrablich Integral Transform

The finite Marchi-Zgrablich integral transform of $f(r)$ is defined as [13]

$$\bar{f}_p(m) = \int_a^b r\, f(r)\, S_p(\alpha,\beta,\mu_m\, r)\, dr \qquad \text{(A1)}$$

Where, α_1, α_2, β_1 and β_2 are the constants involved in the boundary conditions $\alpha_1 f(r) + \alpha_2\ f'(r)\big|_{r=a} = 0$ and $\beta_1 f(r) + \beta_2\ f'(r)\big|_{r=b} = 0$ for the differential equation $f''(r) + (1/r) f'(r) - (p^2/r^2) f(r) = 0$, $\bar{f}_p(m)$ is the transform of $f(r)$ with respect to kernel $S_p(\alpha,\beta,\mu_m\, r)$ and weight function r. The inversion of equation (A1) is given by

$$f(r) = \sum_m \frac{1}{C_m}\ \bar{f}_p(m)\, S_p(\alpha,\beta,\,\mu_m r) \qquad \text{(A2)}$$

Where, kernel function $S_p(\alpha,\beta,\mu_m\, r)$ can be defined as

$$S_p(\alpha,\beta,\mu_m\, r) = J_p(\mu_m r)[G_p(\alpha,\mu_m a) + G_p(\beta,\mu_m b)] - G_p(\mu_m r)[J_p(\alpha,\mu_m a) + J_p(\beta,\mu_m b)] \qquad (36)$$

being

$$J_p(\alpha,\mu_m\xi) = J_p(\mu\xi) + \alpha\,\mu\, J_p{}'(\mu\xi) \quad \text{and} \quad G_p(\alpha,\mu_m\xi) = G_p(\mu\xi) + \alpha\,\mu\, G_p{}'(\mu\xi)\ ,$$

Where, $J_p(\mu r)$ and $G_p(\mu r)$ are Bessel function of first and second kind respectively and

$$C_m = \int_a^b x\{S_p(\alpha,\beta,\,\mu_m x)\}^2\, dx = \frac{b^2}{2}\{S_p^2(\alpha,\beta,\,\mu_m b) - S_{p-1}(\alpha,\beta,\,\mu_m b)\cdot S_{p+1}(\alpha,\beta,\,\mu_m b)\}$$

$$-\frac{a^2}{2}\{S_p^2(\alpha,\beta,\,\mu_m a) - S_{p-1}(\alpha,\beta,\,\mu_m a) S_p(\alpha,\beta,\,\mu_m a)\}$$