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Fractality of Magnetospheric Dynamics: A Diffusion Entropy Approach

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Abstract

There are several self-organization processes in physics such as second-order phase transitions and associated scale-invariant phenomena, structure formation in thermodynamic systems away from equilibrium, self-organization of solitons into vortices in magnetized plasma etc. Space plasmas often display very complex behavior which includes multiscale dynamics, spatio-temporal chaos and self-organized criticality. The study of self-organization in magnetospheric plasma and its relation with instabilities is a subject at the forefront of space research and in particular, having relevance in the analysis of magnetospheric dynamics. The forced self-organized criticality concept was mostly motivated by the physics of magnetospheric substorms, which seems to require a continuous loading process in order to drive it into a critical or near-critical state. Low frequency stochastic fluctuations of the geomagnetic AE index with a $1/f$ *spectrum have been interpreted in terms of a SOC system. We analyze in detail the multifractality of the auroral indices such as AE, AL and AU which may give an insight into the existence of self-organization in the magnetotail with underlying complex multifractal accumulation/dissipation dynamics in the plasma sheet.*

Keywords: *Diffusion entropy, fractal, magnetosphere*

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INTRODUCTION

The earth's magnetosphere is considered as a complex dynamical system driven constantly by the solar wind. Due to this continuous forcing, the magnetotail plasma sheet is driven into a non-equilibrium self-organized global state, characterized by criticalities with scale invariant events, self-similar spatial structures, and multifractal topology [1, 2]. These are similar to out-of-equilibrium states which are seen to emerge naturally in numerous plasma physics models by sporadic dissipation, through spatio-temporal chaos. Scaling behavior in a geomagnetic time series like auroral and polar cap indices represents a typical signature of a multi-time scale cooperative behavior which reflects the underlying long-range cooperative interactions existing in the complex magnetosphere system [3]. It also reveals inner structure of the magnetosphere with which we can draw inferences whether the processes under study are coupled or not. Physical models of the dynamics of the earth's magnetotail are described in terms of stochastic behavior of a nonlinear dynamical system near forced and/or self-organized criticality. Multi-scale intermittent turbulence of overlapping plasma resonances and current-driven instabilities are believed to lead to the onset and evolution of substorms, which explains the localized and sporadic nature of bursty magnetic reconnection and the fractal spectra observed in the magnetotail region.

Numerous studies have considered the fractal and multifractal structure of space physics data [3–5]. A fractal is an object in which the parts are in some way related to the whole. Self-similarity, invariance with respect to scaling, is an important characteristic of fractals. It means that the object or process is similar at different scales. As an example, we may say that individual branches of a tree are qualitatively self-similar to the other branches, but each branch is also unique. A self-similar process is also called uniscaling or monofractal. Multifractal process extends the idea of similarity to generalized scaling (or multiscaling) that includes both long-memory as well as extreme variations [6]. Scaling properties in moments of the process are the most common way to study multifractality. Several time-domain variance-based methods are proposed till date to detect the fractal features, such as the detrended fluctuation approach (DFA), the factorial moments and the detrended moving average (DMA) [7–10]. Similarly, in the frequency-domain also fractal analyses can be carried out using simple methods such as the scaling of power spectral density (PSD) [11]. The major short-coming of the variance-based methods is that albeit the scale-invariance property may be noticed, it is difficult to precisely calculate the value of the exponent. For processes such as Lévy flights, the variance tends to be infinite and accurate results may not be yielded using variance-based methods. Even though, time-frequency wavelet methods such as wavelet transform modulus maxima (WTMM) are proven useful for multifractal analysis, it also suffers certain drawbacks [12]. To ensure reliable results, selection of a well-matched analyzing wavelet is crucial. Also, if the initial and final points of the signal exhibit scaling artifacts, inappropriate selection of scales would affect the accuracy of the results. The diffusion entropy approach (DEA) used in this study can overcome most of these difficulties and by extending ordinary DEA to a generalized multifractal DEA (MF-DEA) approach, it is possible to extract multifractal features of magnetospheric indices [13–19]. Unlike DFA, DEA does not rely on trends superposed on geomagnetic fluctuations while it is based on the direct evaluation of information entropy of the diffusion process. It is well known that entropy is considered as a more comprehensive indicator of the stochasticity inherent in a distribution. The extraction of the temporal scaling exponent of diffusion processes using DEA is valid for both Gaussian (which have finite variance) and non-Gaussian statistics such as heavy-tailed distributions and Lévy processes. This makes DEA or its multifractal variant one of the most promising methods in the study of fractality and self-similarity of geomagnetic indices. Morozov had made some comments regarding the need of a strong mathematical background for MF-DEA technique [17, 18, 20]. Jizba and Korbel have extensively verified the different mathematical aspects of MF-DEA and they have also formulated an efficient method for the computation of probabilities through optimized binning [19, 21].

The aim in this paper is to find multifractal scaling features of auroral and indices during calm and magnetically disturbed periods and to compare the multifractality of AE, AU, AL and PC indices, considering their relationship with solar activity. The changes in the magnetosphere system are evaluated using the diffusion entropy exponent and associated mass exponent which are calculated from aforesaid geomagnetic proxies. The rest of the paper is structured as follows: The data used for the present work are discussed at first; then multifractal diffusion entropy (MF-DEA) technique used to evaluate multifractality is summarized briefly. After this, MF-DEA technique is applied to the aforementioned proxies and lastly, the implications of the results are discussed.

DATA

1 min AE, AU and AL indices were taken from the World Data Center for Geomagnetism, Kyoto, Japan and PC index from the World Data Center for Geomagnetism, Copenhagen, Denmark.

METHOD

If a stationary time series, $\{x(m)\}_{m=1}^{\zeta}$ is considered such that all possible segments with length l could be written as:

$$\psi_i = \{x_i, x_{i+1}, \ldots, x_{i+l-1}\}; i = 1,2,\ldots,\zeta - l + 1 \quad (1)$$

If length l is taken as time scale, then vector Z_i can be regarded as the particle's trajectory beginning from its initial state $\psi_i(0) = 0$. Likewise, actual time series can be mapped into an ensemble which contains $(\zeta - l + 1)$ realizations of a stochastic process:

$$\xi_i(l) = \sum_{m=1}^{l} \psi_i(m) \quad (2)$$

If the displacement interval is partitioned such that the particle appears into $B(l)$ bins and the numbers of particle's occurrences in each bin at time $l$denoted as: $\zeta_m(l); m = 1,2,\ldots,B(l)$, then probability density function (PDF) can be approximated by the relative frequency:

$$P(m,l) = \frac{\zeta_m(l)}{\zeta - l + 1};\ m = 1,2,\ldots,B(l) \quad (3)$$

Now, the diffusion entropy will be:

$$DiffEn = -\sum_{j=1}^{B(l)} P(m,l) log P(m,l) \quad (4)$$

But, in order to analyze fractal-order scaling (with moment of order $q \neq 1, q \in R$) of a multifractal series, the diffusion entropy has to be replaced with its generalized entropy analogue called q-Rényi entropy which is defined as:

$$S_q\ (l)\ = \frac{1}{1-q} log \sum_m\ [P(m,l)]^q \qquad (5)$$

The scaling exponent $\tau(q)$ can be calculated from the linear regression of $S_q(l,t)[\tau(q)logl]$ from the relation:

$$S_q(l,t) = K_q(t) + \tau(q)logl \qquad (6)$$

Where, K_q is a l-independent constant. If a discrete time-series such as $\{x(m)\}_{m=1}^{\zeta}$ is taken and dividing the whole domain of $x(m)$'s into definite regions, where each $x(m)$ are acquired from measurements at times, say $t(m)$, with an equidistant lag l, then for every region, the probability scales as $\mathrm{P}(i,l) \propto l^{\alpha_i}$. Here the scaling exponents α_i are called as the singularity or Lipschitz-Hölder exponents.

The $f(\alpha)$is known as the multifractal spectrum which represents the fractal dimension f of the set of points that corresponds to a Lipschitz-Hölder exponent α. If we consider the two conjugate pairs $\{f(\alpha),\alpha\}$ and $\{\tau(q),q\}$, then the Legendre transform between them yields:

$$\tau(q) = q\alpha(q) - f\big(\alpha(q)\big) \qquad (7)$$

In other words, $\alpha(q)$ is that value of the Lipschitz-Hölder exponent, which maximizes $[q\alpha - f(\alpha)]$for a given q.

Also, the generalized q-order self-similar exponent H_q can be expressed by the relation:

$$qH_q = \tau(q) + 1 \qquad (8)$$

The detailed derivation of various mathematical expressions recalled here is outside the purview of the current study and the reader is referred to the articles of Scafetta and Grigolini as well as Jizba and Korbel and references therein [14, 19, 21].

RESULTS

In the present study, diffusion entropy method has been used for the identification of intermittency of magnetospheric proxies, which display multifractal features of magnetosphere during disturbance as well as calm times, focusing mainly on auroral indices (AE, AU and AL) and PC index. The MF-DEA method mainly relies on the fluctuation collection algorithm and optimized binning of the fluctuations at different scales. Figure 1 shows the fluctuations of 1 min AE index during a sample disturbance period (a period of strong geomagnetic storm on August 10–14, (2000) collected at different scales.

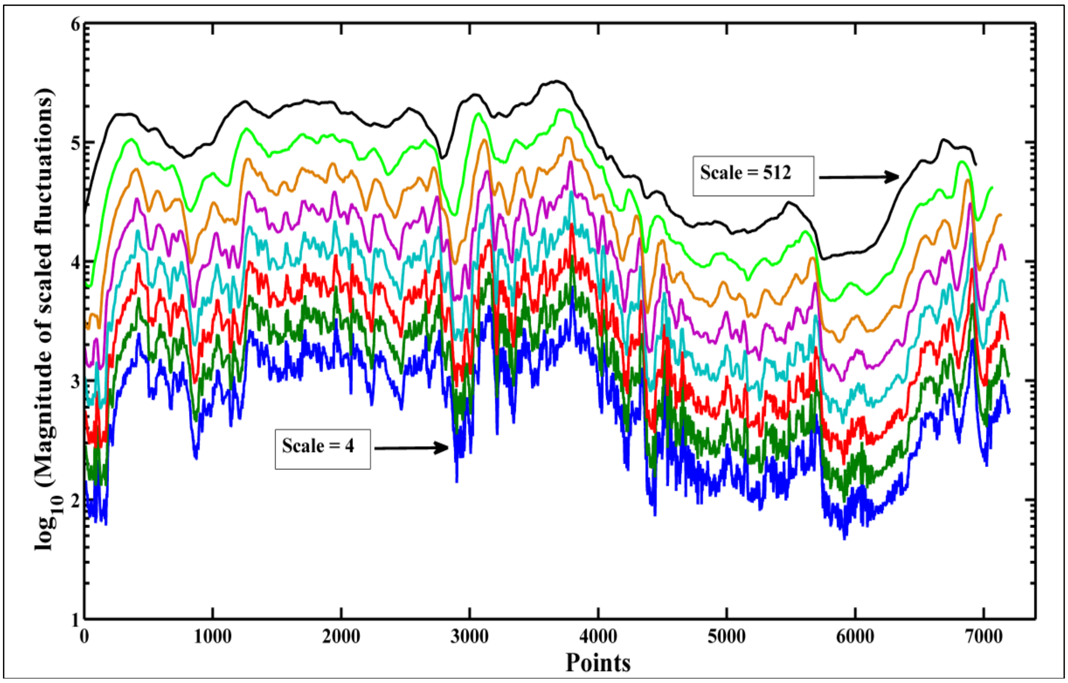

Fig. 1: *Magnitude of Scaled Fluctuations of AE Index against Points of the Time Series with Lowest Scale is* 2^2 *and Highest Scale is* 2^9*. The Sample Event Considered Here is the Geomagnetic Disturbance Period during August 10–14 (2000).*

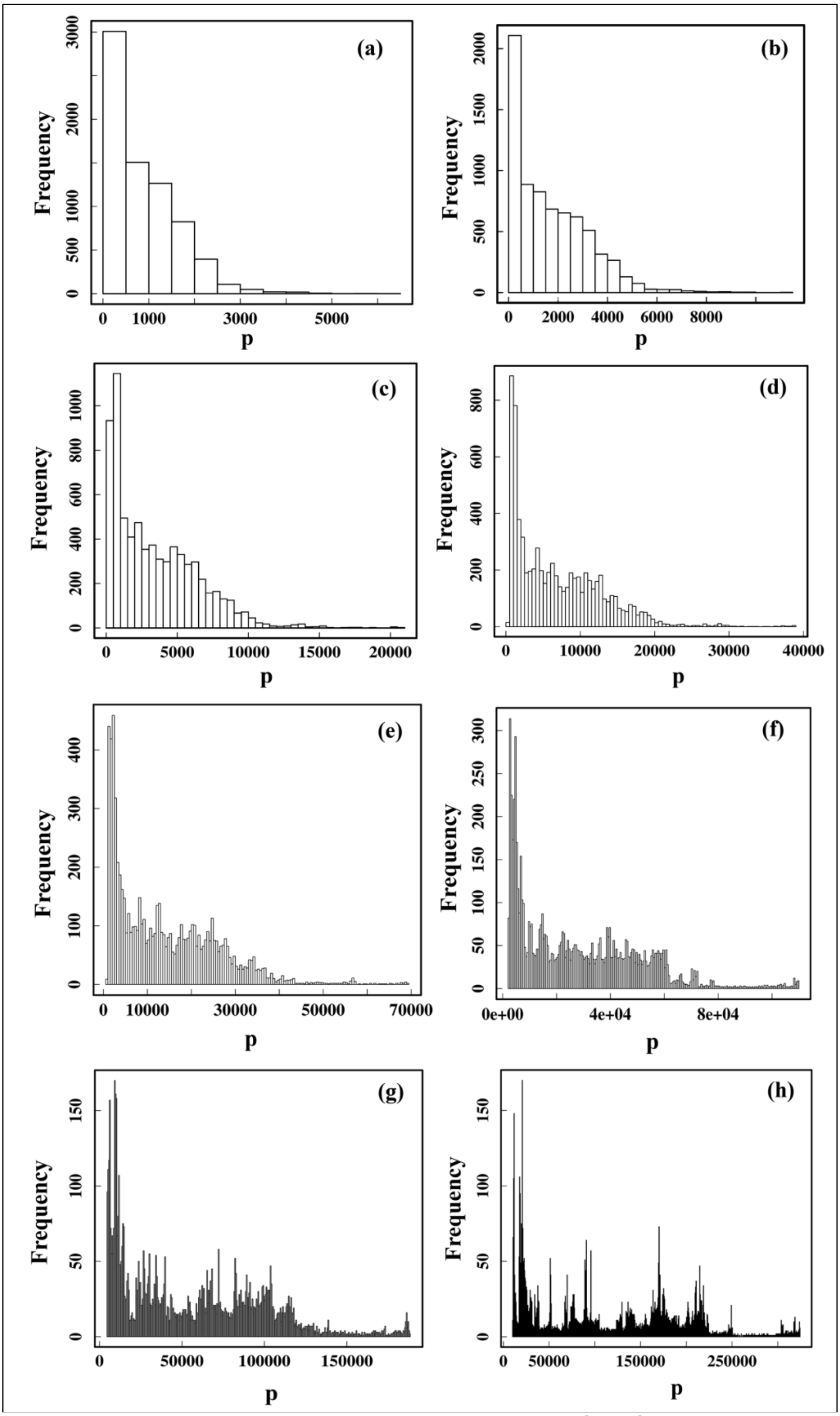

Fig. 2: *Frequency Histograms Corresponding to the Scales* 2^2 *to* 2^9 *Shown in Figure 1.*

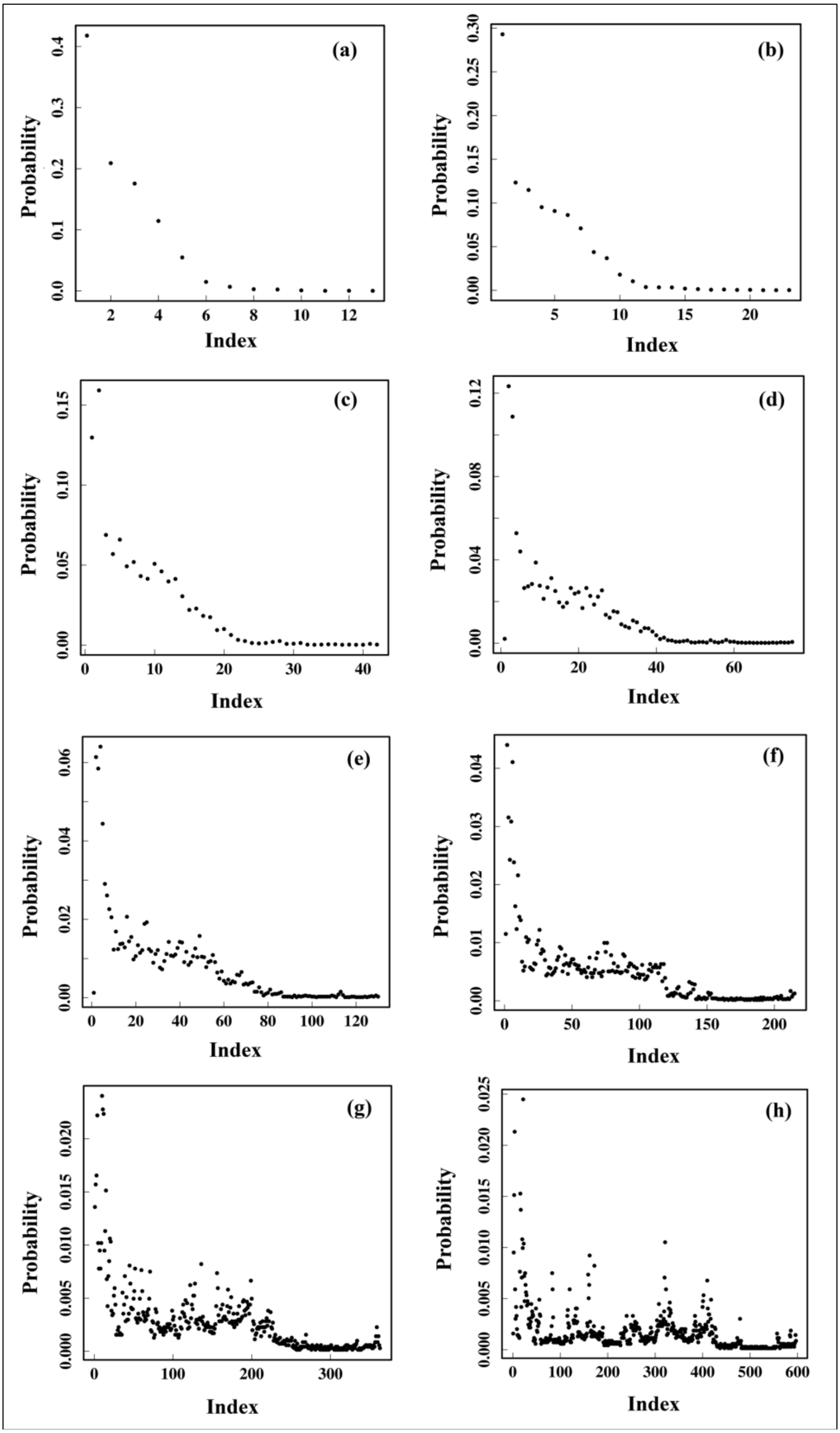

Fig. 3: *Probabilities Computed from the Frequency Histograms of Figure 2.*

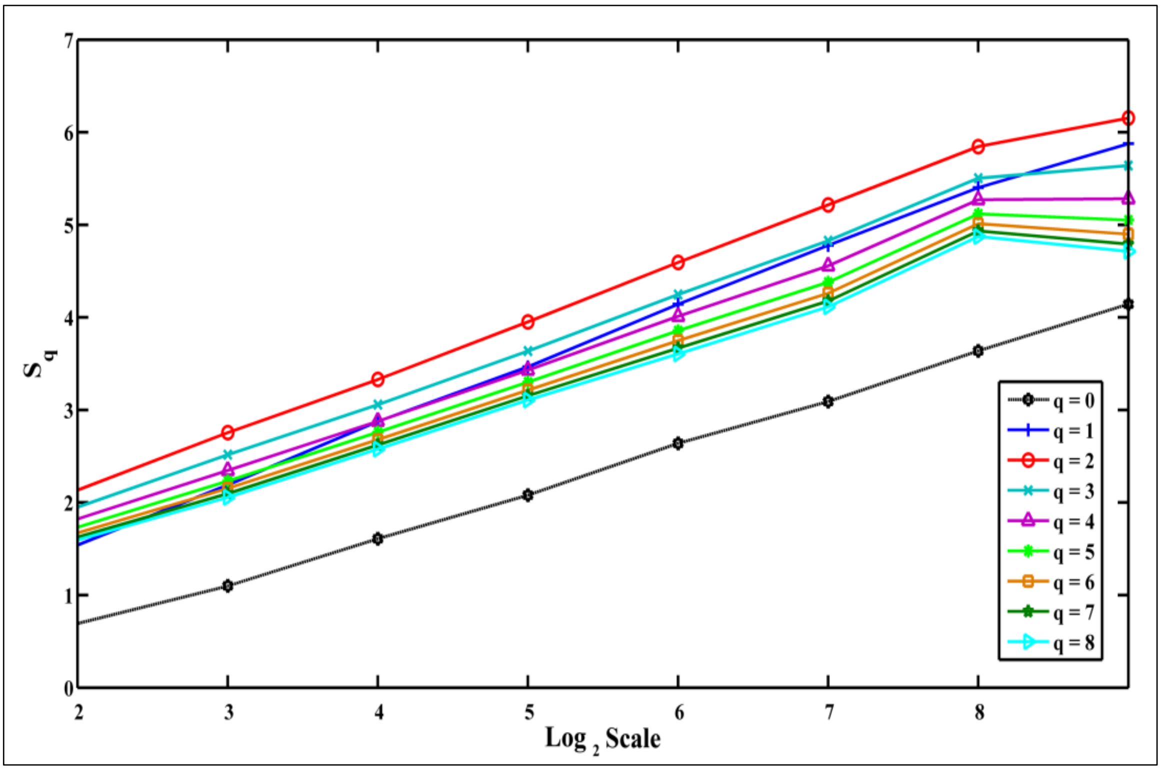

Fig. 4: *Diffusion Entropy versus Scale for q in the Range of 0 and 8 for AE Index during August 10–14 (2000).*

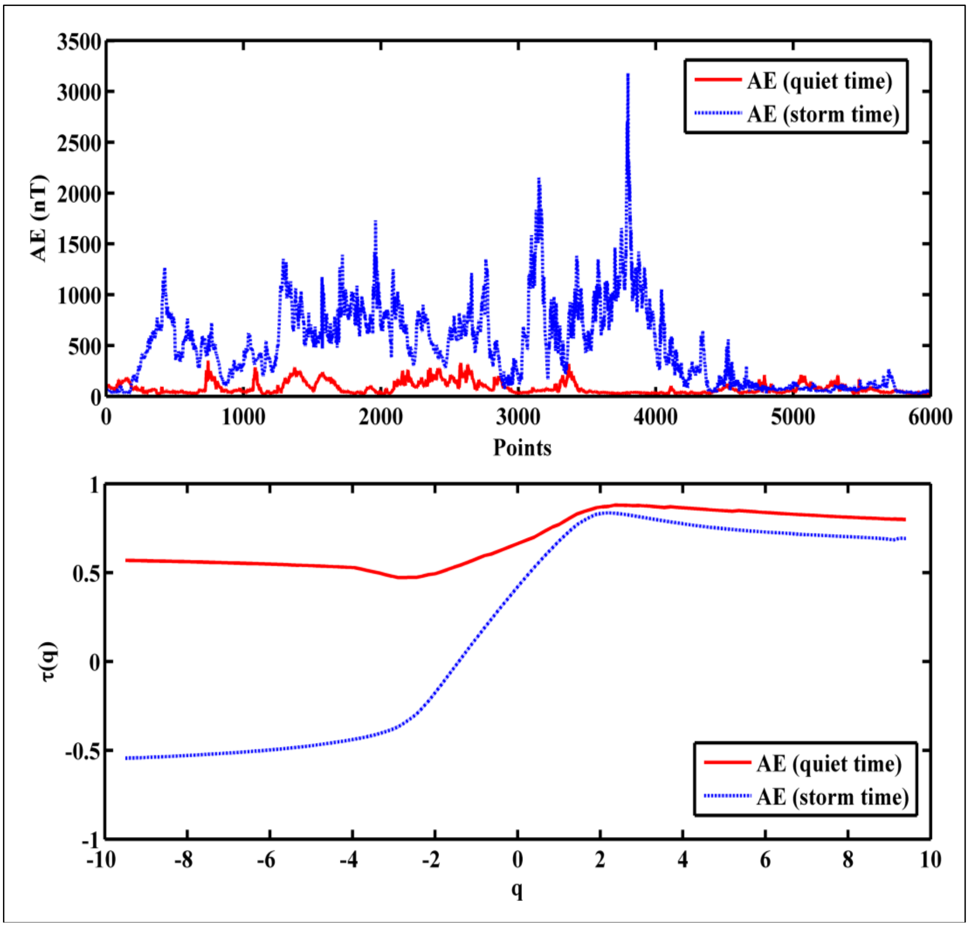

Fig. 5: *Variation of Mass Exponent Spectrum of AE Index during Quiet and Storm Periods.*

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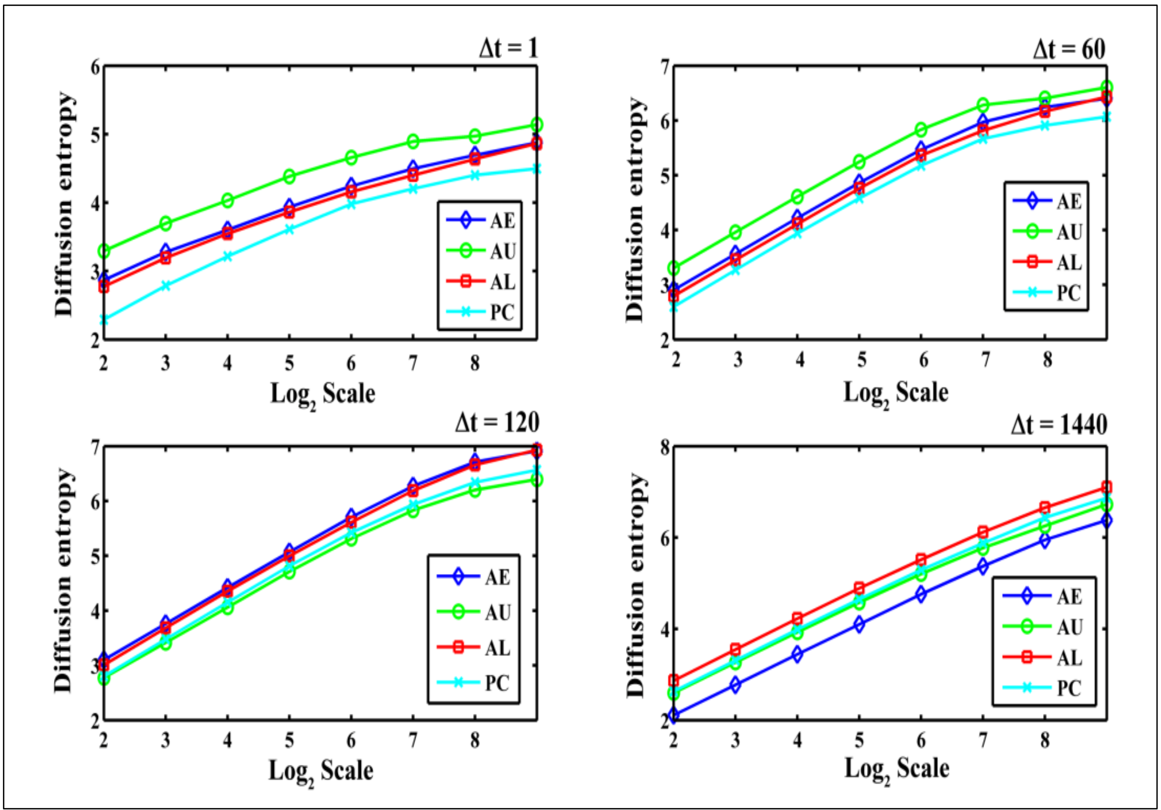

Fig. 6: *The Changes in Diffusion Entropy for q=1 Taking AE, AU, AL and PC Indices. The Δt Values Considered are 1, 60, 120, 1440.*

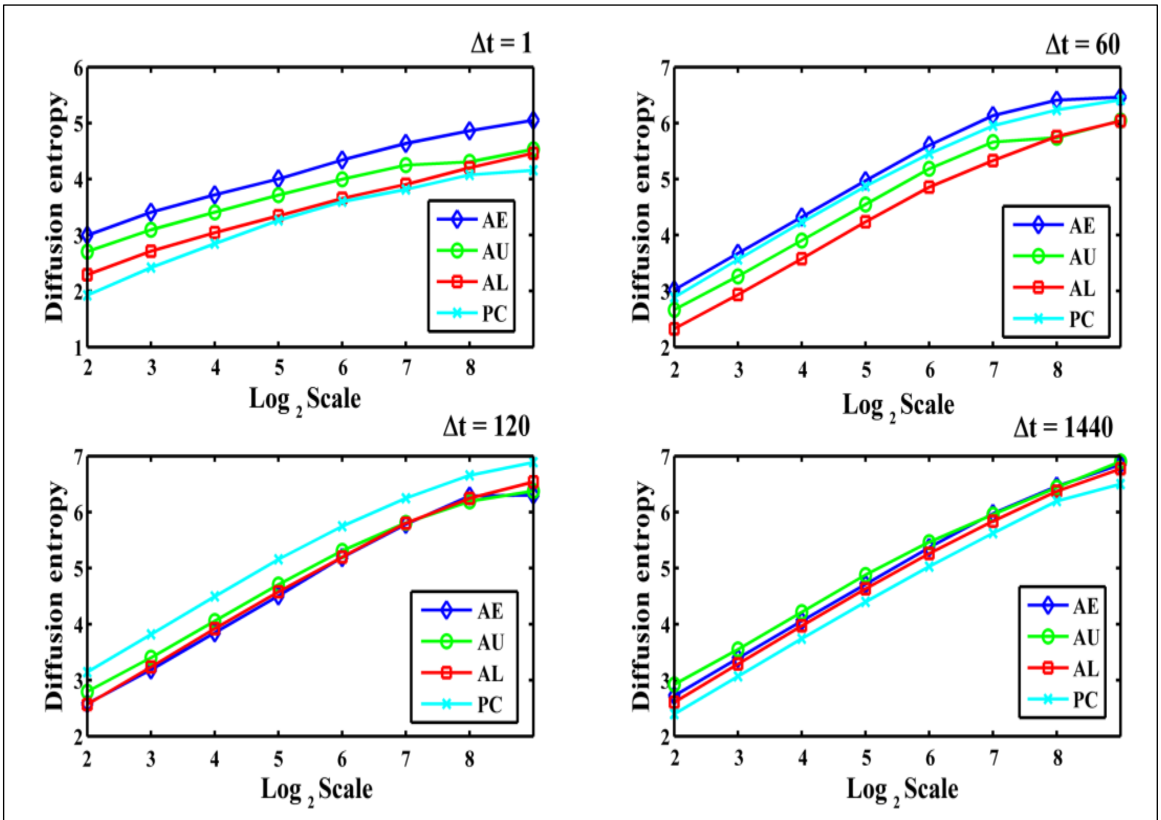

Fig. 7: *"The Changes in Diffusion Entropy for q=2 Taking AE, AU, AL and PC Indices. The Δt Values Considered are 1, 60, 120, 1440".*

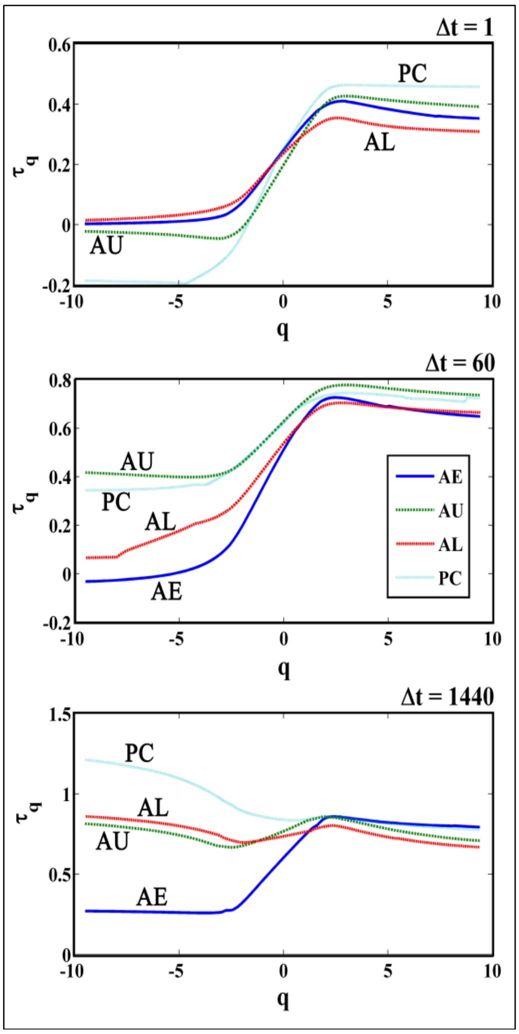

Fig. 8: *The Mass Exponent Spectra of AE, AU, AL and PC for Δt values 1, 60, 120, 1440.*

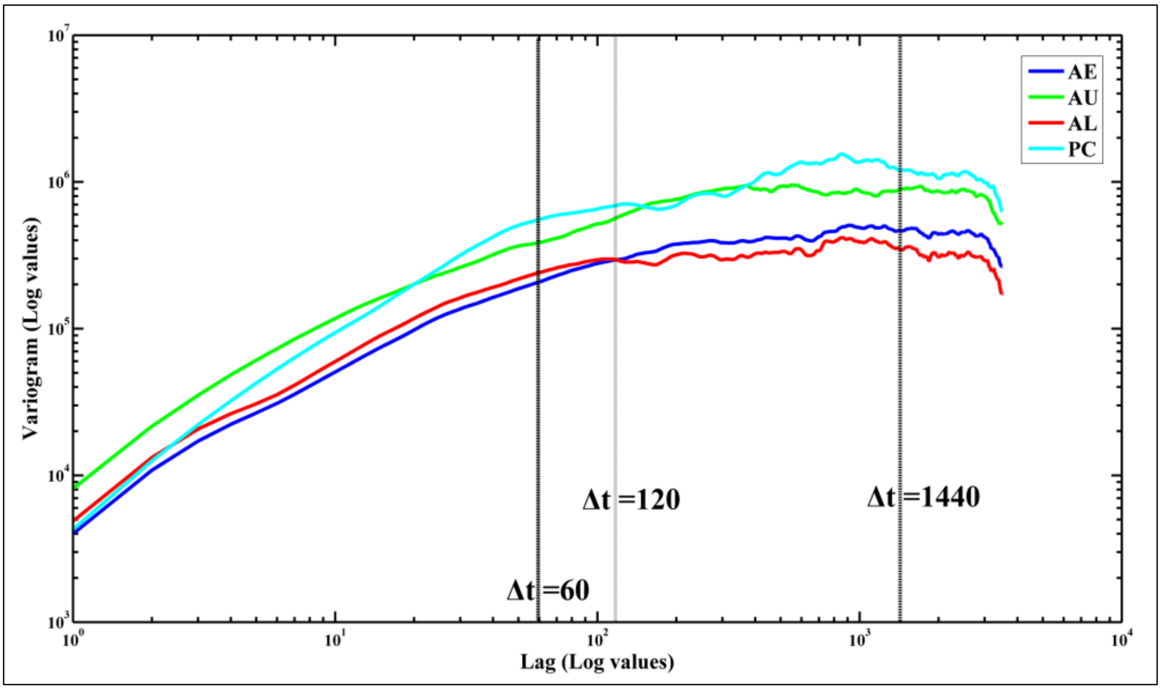

Fig. 9: *The Variogram of AE, AU, AL and PC for Δt Values 1, 60, 120, 1440.*

Similarly, the fluctuations of other auroral indices at both disturbance and calm periods can also be collected in the same procedure. Frequency histograms of fluctuations collected from AE index at scales 2^2 to 2^9 are shown in Figure 2. Figure 3 shows probabilities computed from the frequency histograms for AE index. Diffusion entropy curves against scales for q in the range 0 to 8 for AE index are shown in Figure 4. Now for a comparison in the variation of mass exponent spectra, in Figure 5, the curves during storm and calm times have been shown.

As evident from the Figure 5, the mass exponent spectrum varies drastically between calm and disturbance times. The width of the spectrum (as defined by the maximum and minimum values of τ) increases significantly during disturbance times. This clearly shows that MF-DEA successfully tracks the variation of magnetospheric dynamics during calm and disturbance times. The results go well with the earlier results of Uritsky and Pudovkin as well as Rypdal and Rypdal who gave evidence suggesting that the behavior of AE and solar wind quantities is not just self-similar but multifractal [20, 22].

Now, if a new time series is created based on differencing of the original time series over a range of temporal scales Δt such that $\delta x(t, \Delta t) = x(t + \Delta t) - x(t)$, it will capture fluctuations on temporal scale Δt [23]. The differencing is performed only if both $x(t + \Delta t)$ and $x(t)$ exist and are separated by time interval Δt. For q=1 and 2, we consider four Δt values such as 1, 60, 120 and 1440 corresponding to 1 min, 1 h, 2 h and 1 day. Figures 6 and 7 represent the scaling of auroral and polar cap indices for the geomagnetic storm time which clearly indicate that multifractality reduces for Δt =120 and the series almost turns monofractal for Δt =1440.

Figure 8 shows the variation of corresponding variations in the mass exponent spectra of AE, AU, AL and PC index. Here also, consistent with previous studies, self-similar scaling in the geomagnetic indices on timescales shorter than 1–2 h, that is, shorter than the characteristic substorm timescale has been found [24–26]. The proper choice of temporal scales could thus naturally includes/excludes substorm characteristics from magnetic storm features. Figure 9 shows the variogram of AE, AU, AL and PC indices where it is evident that the slope varies with scaling regions.

CONCLUSION

A novel method called MF-DEA has been used for the identification of scaling, self-similarity and multifractality. MF-DEA approach reveals the multifractal characteristics of magnetosphere during geomagnetic disturbance times and calm times. The study shows that diffusion entropy could successfully identify the dynamical fingerprints during geomagnetically disturbance periods and calm periods. The diffusion entropy analysis shows that there exists a completely different scaling in auroral and PC indices while comparing ranges of the order of one day or more with substorm time scales (<120 min). The magnetospheric plasma system is strongly coupled externally to the solar wind plasma flow revealing dissipative internal non-equilibrium and nonlinear dynamics. Until now, main theme of magnetospheric dynamics remains the development of magnetospheric superstorms during which, strong plasma flows can be developed along the magnetotail. The response of the earth's magnetosphere to the solar wind illuminates the interplay between intrinsic magnetospheric dynamics and solar wind-magnetosphere coupling. In the context of intermittent analysis, diffusion entropy based studies provide a simple yet unifying way to quantify this behavior.

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