

Potential Energy Surface for $^{190-198}\text{Hg}$ Isotopes

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Abstract

The $^{190-198}\text{Hg}$ isotopes with proton number $Z=80$ and neutron numbers (n) between 110 and 118 in $O(6)$ region were investigated. The potential energy surface has been calculated within the framework using interacting boson model (IBM-1). The contour plot of the potential energy surfaces shows that the interested nuclei are deformed and have γ -unstable-like characters.

Keywords: IBM-1, $O(6)$, Hg isotopes, Potential energy surface

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INTRODUCTION

The nuclear structure of the mercury region has proven difficult to interpret in terms of the traditional descriptions [1]. These nuclei were characterized by shape changes between spherical and deformed [2]. The IBM-1 has a group structure $U(6)$. The three limiting symmetries of this Hamiltonian, $U(5)$, $SU(3)$, and $O(6)$, correspond to the geometrical shapes, spherical vibrator, symmetric rotor, and γ -unstable rotor, respectively [2].

In calculations [3,4] within the Interacting Boson Model (IBM-1), mercury nuclei have been successfully treated as exhibiting the $O(6)$ symmetry [5] of this model. The IBM [6] has been successful in reproducing the nuclear collective levels in terms of s and d bosons, which are essentially the collective s and d pairs of valence nucleons with angular momentum $L=0$ and 2 [7].

The application of this model to deformed nuclei is currently subject of considerable interest and controversy. Here, we apply the IBM model to account for even-even mercury isotopes. Detailed work has been done on the structure of mercury nuclei in recent years. Gall et al. [8] studied the rotational properties of the ground super-deformed bands in ^{190}Hg , ^{192}Hg , ^{194}Hg , and ^{194}Pb by use the cranked Hartree-Fock-Bogoliubov method with the SkM* parametrization of the Skyrme force in the particle-hole channel and a seniority interaction in the pairing channel. Weissman et

al. [9] measured the average g factors of high spin, high-excitation energy, quasi-continuum structures in $^{194,193}\text{Hg}$ by observing the precessions of the angular distributions of g -ray transitions in several normal-deformation bands that coalesce in the decay of the entry distribution of states. Nomura et al. [10] applied the interacting boson model with configuration mixing, with parameters derived from the self-consistent mean-field calculation employing the microscopic Gogny energy density functional, to the systematic analysis of the low-lying structure in Hg isotopes.

Bernards et al. [11] studied 0^+ states in ^{198}Hg after the $^{200}\text{Hg}(p, t)^{198}\text{Hg}$ transfer reaction up to 3 MeV excitation energy and the experiment was performed using the high-resolution Q3D magnetic spectrograph at the Maier-Leibnitz Laboratory Tandem accelerator in Munich.

García-Ramos et al. [12] description the even-even Hg isotopes, $^{172-200}\text{Hg}$, using the interacting boson model including configuration mixing and pay special attention to the description of the shape of the nuclei and to its connection with the shape coexistence phenomenon. Bree et al. [13] examined Coulomb-excitation experiments to electromagnetic properties of radioactive even-even Hg nuclei were performed with 2.85 MeV/nucleon mercury beams from REX-ISOLDE and extracted the magnitudes and relative $B(E2)$ values that couple the

ground state and excited levels in 182–188Hg isotopes. The bulk and decay properties, including deformation energy curves, charge mean square radii, Gamow-Teller (GT) strength distributions, and β -decay half-lives in neutron-deficient even-even and odd-A Hg and Pt isotopes described by Boillos et al. [14]. The nuclear structure is described microscopically from deformed quasi particle random-phase approximation calculations with residual interactions in both particle-hole and particle-particle channels, performed on top of a self-consistent deformed quasi particle Skyrme-Hartree-Fock basis.

Recently, in the same region of Hg isotopes it has studied the evolution properties of the yrast states and the electromagnetic reduced transition probabilities for even-even pt isotopes [15–18] within the frame work of interesting boson model.

INTERACTING BOSON MODEL (IBM-1)

In medium and heavy even-even nuclei, the Interacting Boson Model (IBM-1) is expected that low-lying states far from magic numbers were commanded by excitations of the valence protons and neutrons (i.e., particles outside the magic numbers 2, 8, 20, 28, 50, 82, and 126) just, while the magic numbers is inert. Besides, it is expected that the particle configurations which are coupled (indistinguishable particles) together forming pairs of angular momentum 0 and 2.

Likewise, these proton (neutron) pairs are dealt as bosons. Proton (neutron) bosons with angular momentum $L = 0$ are signified by s_π (s_ν) and are called s-bosons, while proton (neutron) bosons with angular momentum $L = 2$ are signified by d_π (d_ν) and are called d-bosons.

The underlying structure of the six dimensional unitary group U(6) of the model leads to a simple Hamiltonian, capable of describing the three specific types of collective structure with classical geometrical analogs vibrational SU(5), rotational SU(3), and γ -unstable O(6)[19, 20, 21]. Hamiltonian H can be written explicitly in terms of boson creation (d^\dagger) and annihilation (\tilde{d}) operators [20, 22] such that,

$$\begin{aligned} H = & \varepsilon_s (s^\dagger \cdot \tilde{s}) + \varepsilon_d (d^\dagger \cdot \tilde{d}) \\ & + \sum_{L=0,2,4} \frac{1}{2} (2L+1)^{\frac{1}{2}} C_L \left[[d^\dagger \times d^\dagger]^{(L)} \times [\tilde{d} \times \tilde{d}]^{(L)} \right]^{(0)} \\ & + \frac{1}{\sqrt{2}} v_2 \left[[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)} + [d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)} \right]^{(0)} \\ & + \frac{1}{2} v_0 \left[[d^\dagger \times d^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)} + [s^\dagger \times s^\dagger]^{(0)} \times [\tilde{d} \times \tilde{d}]^{(0)} \right]^{(0)} \\ & + \frac{1}{2} u_0 \left[[s^\dagger \times s^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)} \right]^{(0)} + u_2 \left[[d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)} \right]^{(0)} \end{aligned} \quad (1)$$

This Hamiltonian contains two terms of one body interactions, (ε_s and ε_d), and seven terms of two-body interactions [c_L ($L = 0, 2, 4$), v_L ($L = 0, 2$), u_L ($L = 0, 2$)], where, ε_s and ε_d are the single-boson energies, and c_L , v_L and u_L describe the two boson interactions. However, it turns out that for a fixed boson number N , only one of the one-body terms and five of the two body are terms independent, as it can be seen by noting $N = n_s + n_d$ [22].

RESULTS AND DISCUSSION

In Hg nuclei with proton = 80 have neutron holes numbers 8, 7, 6, 5 and 4, and proton hole 1 according to framework of interaction boson model-1 (IBM-1). The total numbers of boson number are 9, 8, 7, 6 and 5 of ¹⁹⁰⁻¹⁹⁸Hg nuclei. The calculated results can be discussed separately for potential energy surface ($E(N, \beta, \gamma)$).

The potential energy surface gives a final shape to the nucleus that corresponds to the function of Hamiltonian [23], as the equation [21, 22]:

$$E(N, \beta, \gamma) = \frac{\langle N, \beta, \gamma | H | N, \beta, \gamma \rangle}{\langle N, \beta, \gamma | N, \beta, \gamma \rangle} \quad (2)$$

The expectation value of the IBM-1 Hamiltonian with the coherent state ($|N, \beta, \gamma\rangle$) is used to create the IBM energy surface [21,22].

The state is a product of boson creation operators (b_c^\dagger), with

$$|N, \beta, \gamma\rangle = \frac{1}{\sqrt{N!}} (b_c^\dagger)^N |0\rangle \quad (3)$$

$$b_c^\dagger = (1 + \beta^2)^{-1/2} \{s^\dagger + \beta [\cos \gamma (d_0^\dagger) + \sqrt{1/2} \sin \gamma (d_2^\dagger + d_{-2}^\dagger)]\} \quad (4)$$

The energy surface, as a function of β and γ , has been given by [22]

$$E(N, \beta, \gamma) = \frac{N \epsilon_d \beta^2}{1 + \beta^2} + \frac{N(N-1)}{(1 + \beta^2)^2} (\alpha_1 \beta^4 + \alpha_2 \beta^3 \cos 3\gamma + \alpha_3 \beta^2 + \alpha_4) \quad (5)$$

where, the α_i 's are related to the coefficients C_L , v_2 , v_0 , u_2 and u_0 of Eq. (1) and β is a measure of the total deformation of nucleus, where $\beta = 0$ the shape is spherical, and is distorted when $\beta \neq 0$, and γ is the amount of deviation from the focus symmetry and correlates with the nucleus, if $\gamma = 0$ the shape is prolate, and if $\gamma = 60$ the shape becomes oblate.

In the Figure 1, the contour plots in the γ - β plane resulting from $E(N, \beta, \gamma)$ are shown for $^{190-198}\text{Hg}$ isotopes. For most of the considered Hg nuclei the mapped IBM energy surfaces are triaxial shape.

Triaxial shape is associated with intermediate values $0 < \gamma < \pi/3$. The triaxial deformation helps to understand the prolate-to-oblate shape transition that occurs in the considered Hg isotopes. The Hg nuclei considered here do not display any rapid structural change but remain γ -soft. This evolution reflects the triaxial deformed as one approaches the neutron shell closure $N = 126$.

Figure 2 shown the Potential Energy Surface $E(N, \beta, \gamma)$ as a function of the deformation parameter β with $\gamma = 0$ and 60 for $^{190-198}\text{Hg}$ isotopes. From this figure, the symmetry between the potential energy surface at $\gamma = 0$ and 60 refer to the Hg nuclei are deformed and it have the γ -unstable shape.

CONCLUSION

The $^{190-198}\text{Hg}$ isotopes with proton number $Z=80$ and neutron numbers (n) between 110 and 118 in O(6) region were investigated. The potential energy surface has been calculated within the framework using interacting boson model (IBM-1). The contour plot of the potential energy surfaces shows that the interested nuclei are deformed and have γ -unstable-like characters.

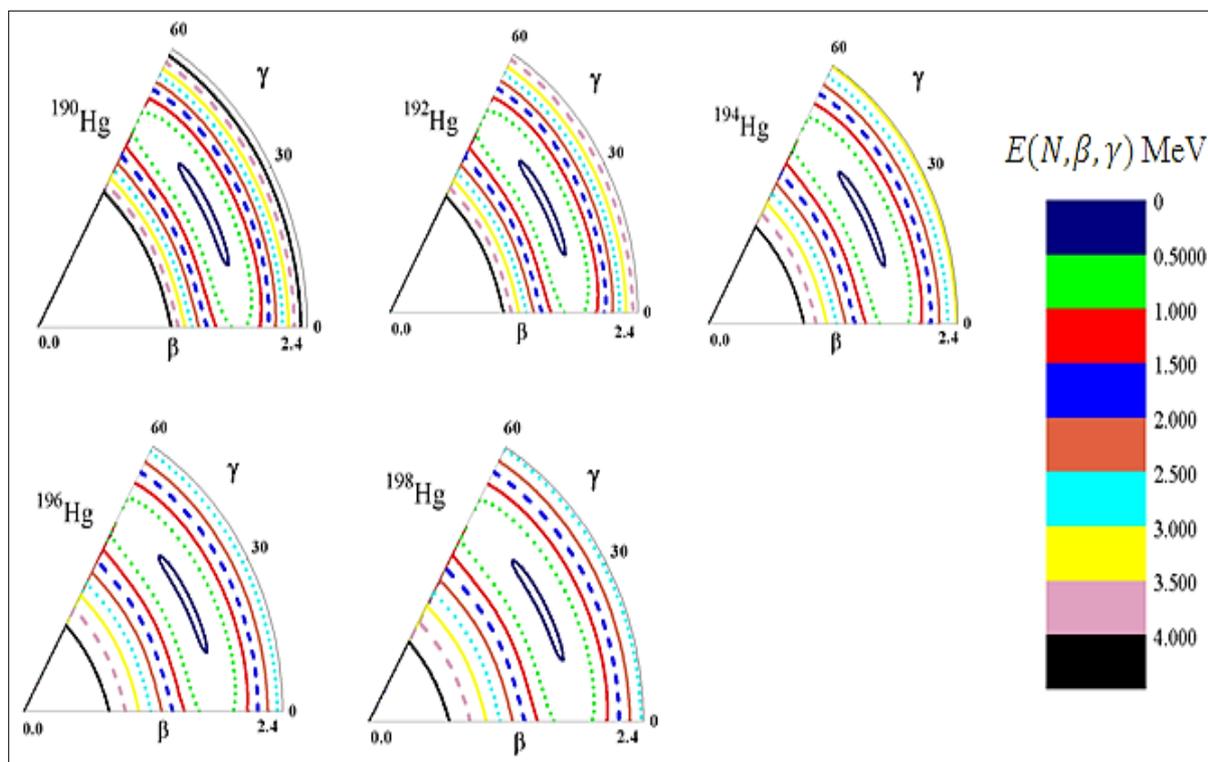


Fig. 1: (Color online) the Potential Energy Surface in γ - β Plane for $^{190-198}\text{Hg}$ Nuclei.

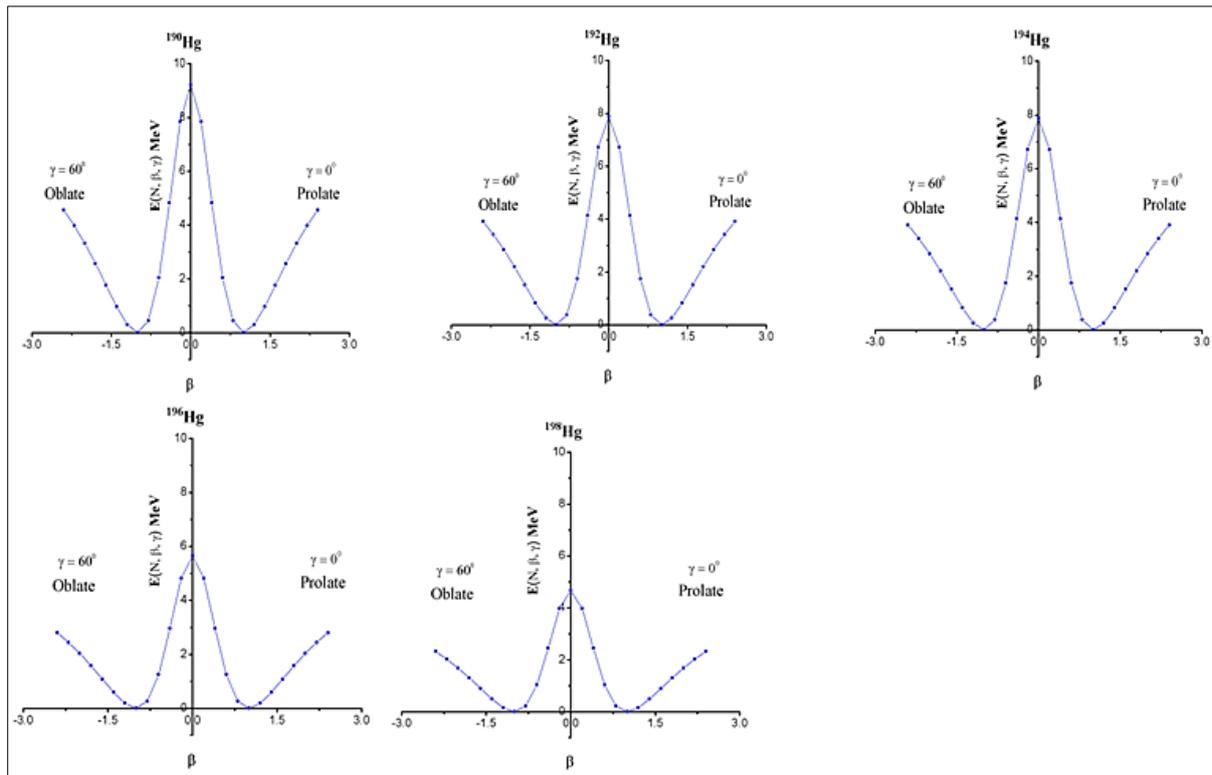


Fig. 2: (Color Online) the Potential Energy Surface $E(N, \beta, \gamma)$ as a function of the Deformation Parameter for $^{190-198}\text{Hg}$ Isotopes.

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