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Anti-synchronization of Bhalekar–Gejji Chaotic System via Nonlinear Active Control

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Abstract
In this paper, a scheme of global anti-synchronization between two identical newly developed Bhalekar–Gejji chaotic system is proposed. This anti-synchronization scheme is achieved by using nonlinear active control since the parameters of both the systems are known and states are measurable. Lyapunov stability theory is used to ensure stability of error dynamics. Controller is designed by using the sum of relevant variables in master and slave chaotic systems. Simulation results reveal that proposed scheme is working satisfactorily.

***Keywords**: Bhalekar–Gejji chaotic system, synchronization, anti-synchronization, nonlinear active control, lyapunov stability*

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INTRODUCTION

Dynamical systems are generally determined by its initial conditions (deterministic) but whose evolution cannot be predicted in the long-term is known as chaotic dynamical system. The field was created in 1970s by the work shown by Lorenz (convection in atmosphere), Ruelle and Takens (turbulence), May (dynamics of populations), Feigenbaum (connection between chaos and phase transition), Winfree (nonlinear oscillations in biology), etc. Due to its random, unpredictable and interesting properties, chaotic system synchronization [1, 2] has been an emerging field of interest in the nonlinear control. In recent years, chaotic system has been used in many fields such as electrical system [3–6], ecological system [2], semiconductor laser system [7] in quantum physics, secure communication [8, 9], complex dynamical networks [10], etc.

The most familiar synchronization phenomenon is that the difference of states of synchronized systems converges to zero, and is called complete synchronization (CS). Almost all the research work reports on chaotic synchronization are relevant to CS. Meanwhile, many chaotic system oscillators have been studied in recent year for anti-synchronization. The sufficient condition is derived for stability of error dynamics. Controllers are designed by using sum of relevant variables in chaotic systems such as Colpitts [11], Lu and Rossler [12], Lorenz and Lu, Lorenz and Chen [13], Genesio and Rossler [14], hyperchaotic Lu and hyperchaotic Chen [15], two different hyperchaotic systems as Chen and Lu [16], identical Lu-Lu and Lorenz-Lorenz, nonidentical Lu and Lorenz [17], Li and Cai [18], Li and Lu [19], Li and T [20], Liu and Chen [21], Pan and Liu [22], etc. This paper presents anti-synchronization of newly developed Bhalekar–Gejji chaotic system [23]. This dynamical system was proposed by Bhalekar and Gejji (Bhalekar & Daftardar-Gejji, 2011; Bhalekar, 2012), known as Bhalekar–Gejji dynamical/chaotic system. Nonlinear active control is used for anti-synchronization between master and slave systems. Identical Bhalekar–Gejji chaotic systems are used as master and slave systems. Convergence and stabilization of error dynamics is achieved using the Lyapunov's stability theory.

The paper describes Bhalekar–Gejji chaotic system, design of nonlinear active control for anti-synchronization of two identical Bhalekar–Gejji chaotic systems. Stability analysis using proposed control design is presented followed by results and discussion for the validation and verification of proposed control scheme. Finally summary of paper is derived as conclusions.

SYSTEM DESCRIPTION AND PROBLEM FORMULATION

As per Ref. [23], the dynamical system proposed by Bhalekar and Gejji is known as Bhalekar–Gejji dynamical system [23]. Here the problem is to anti-synchronize identical chaotic systems with appropriate control. The differential equation of 3D Bhalekar–Gejji chaotic system is described in Eq. (1). A chaotic behavior for $\omega = -2.667$, $\mu = 10$, $\alpha = 27.3$, $b = 1$ is shown. Phase space behavior is given in Figure 1.

$$\begin{aligned}\dot{x}_1 &= wx_1 - x_2^2 \\ \dot{x}_2 &= \mu(x_3 - x_2) \\ \dot{x}_3 &= \alpha x_2 - bx_3 + x_1x_2\end{aligned} \tag{1}$$

Where, x_1, x_2, x_3 are states. $\omega = -2.667$, $\mu = 10$, $\alpha = 27.3$, $b = 1$ are the parameters of Eq. (1).

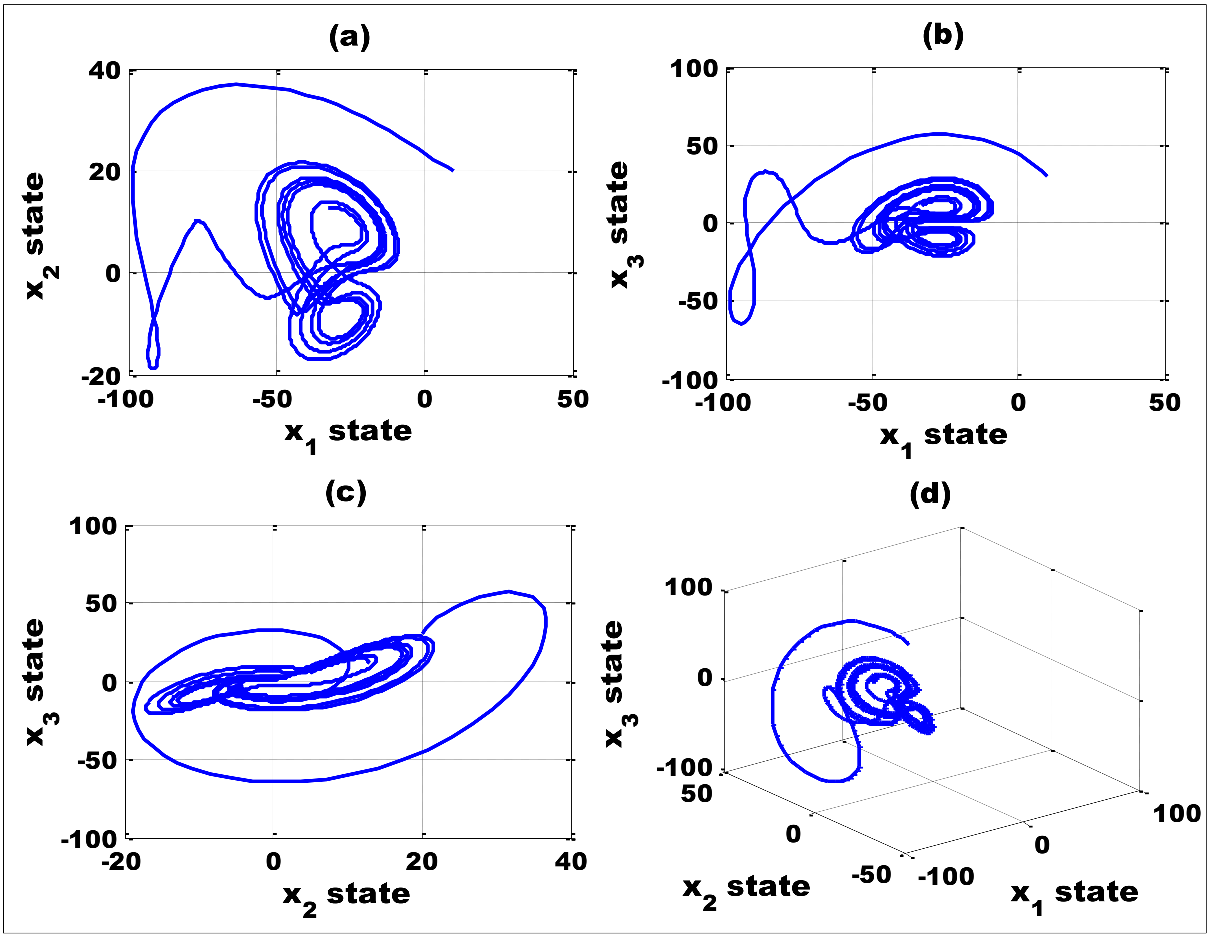

***Fig. 1:** Phase Space Behavior of Bhalekar–Gejji Chaotic System.*

Controller Design for Synchronization of Identical Bhalekar–Gejji Chaotic System

The slave system with control law is described as:

$$\begin{aligned}\dot{y}_1 &= wy_1 - y_2^2 + u_1 \\ \dot{y}_2 &= \mu(y_3 - y_2) + u_2 \\ \dot{y}_3 &= \alpha y_2 - by_3 + y_1y_2 + u_2\end{aligned} \tag{2}$$

Where, y_1, y_2, y_3 are the states of the slave system Eq. (2) and u_1, u_2 u_3 are the control input in the slave system. Our aim is to design nonlinear active control law. Therefore, using Eqs. (1) and (2), error dynamics equation is obtained as follows:

$$\begin{aligned}\dot{e}_1 &= \dot{y}_1 + \dot{x}_1 = we_1 - y_2^2 - x_2^2 + u_1 \\ \dot{e}_2 &= \dot{y}_2 + \dot{x}_2 = -\mu\, e_2 + \mu\, e_3 + u_2 \\ \dot{e}_3 &= \dot{y}_3 - \dot{x}_3 = \ ae_2 - be_3 + y_1y_2 + x_1x_2 + u_3\end{aligned} \tag{3}$$

Designer needs to satisfy following conditions to anti-synchronize the chaotic systems defined in Eqs. (1) and (2)

i.e., $\lim_{t\to\infty}\|\boldsymbol{e}(t)\| = 0 \quad \forall\, \boldsymbol{e}(t)\epsilon R^n$

Let the nonlinear active control law is defined as:

$$\begin{aligned}u_1 &= y_2^2 + x_2^2 \\ u_2 &= -\mu e_3 \\ u_3 &= -ae_2 - y_1y_2 - x_1x_2\end{aligned} \tag{4}$$

Substituting Eq. (4) into Eq. (3), error dynamics can be written as:

$$\begin{aligned} \dot{e}_1 &= we_1 \\ \dot{e}_2 &= -\mu e_2 \\ \dot{e}_3 &= -be_3 \end{aligned} \quad (5)$$

After the design of controller, stability analysis is discussed in the next section.

Stability Analysis

***Theorem**:* The identical Bhalekar–Gejji systems in Eqs. (1) and (2) are globally and exponentially anti-synchronized using nonlinear active control law as given in Eq. (4).

Proof: To establish stability and convergence of error dynamics using Lyapunov theory [24] considering the positive definite Lyapunov function on R^3as;

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \quad (6)$$

Assuming first order partial time derivative of Eq. (6) as;

$$\dot{V}(e) = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \quad (7)$$

$$\dot{V}(e) = we_1^2 - \mu e_2^2 - be_3^2 \quad (8)$$

Eq. (8) is negative definite function on R^3 because $w < 0$. Thus, according to Lyapunov stability theory, an anti-synchronization error dynamic as given in Eq. (3) is found to be asymptotically stable and converges to zero exponentially with time. Therefore, identical Bhalekar–Gejji systems in Eqs. (1) and (2) are globally and exponentially anti-synchronized via nonlinear active control law as Eq. (4). Results and discussions are given in the next section for the proposed controller and the states response of master and slave systems.

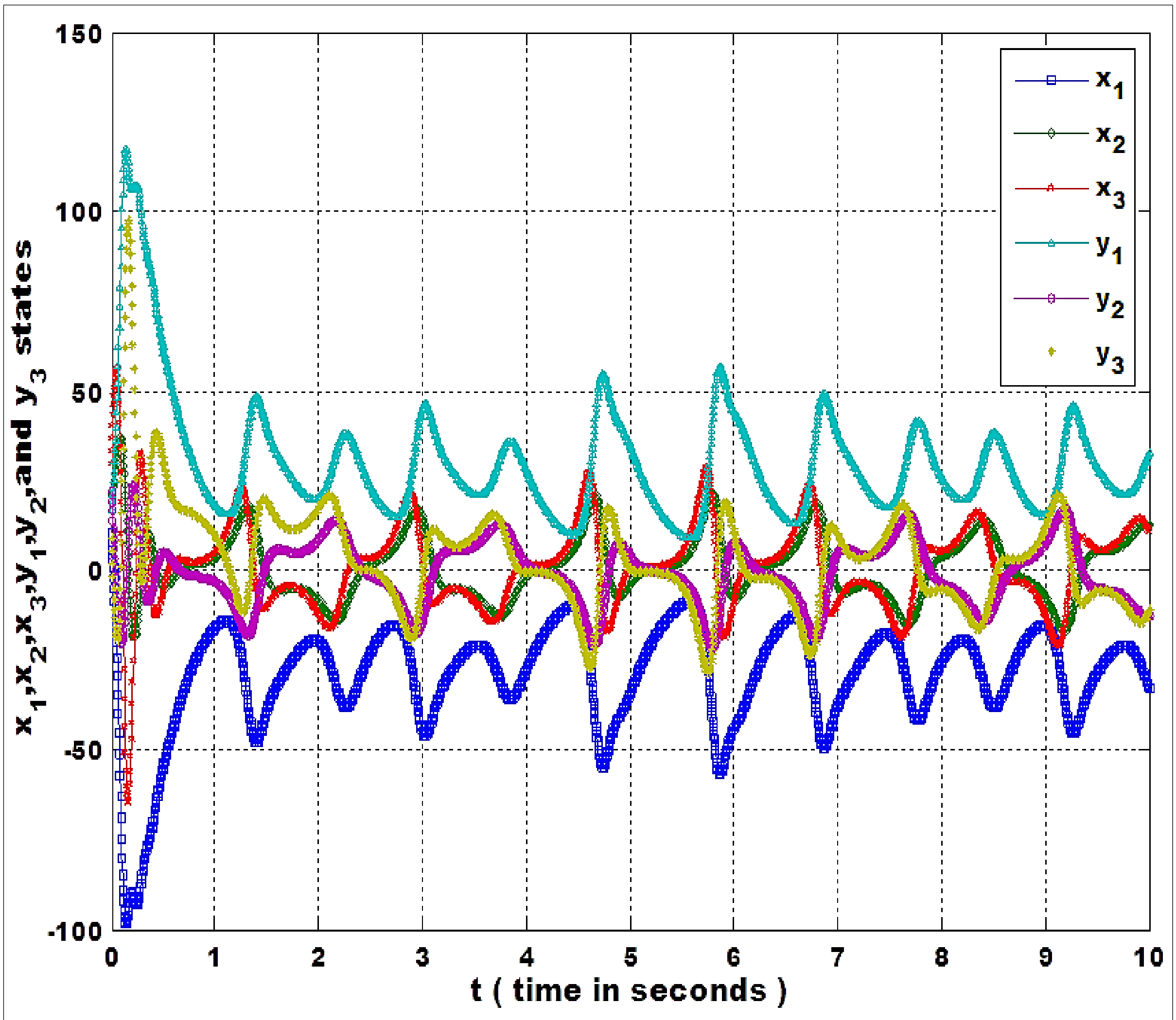

Fig. 2: *State Response of Master (1) and Slave Systems (2).*

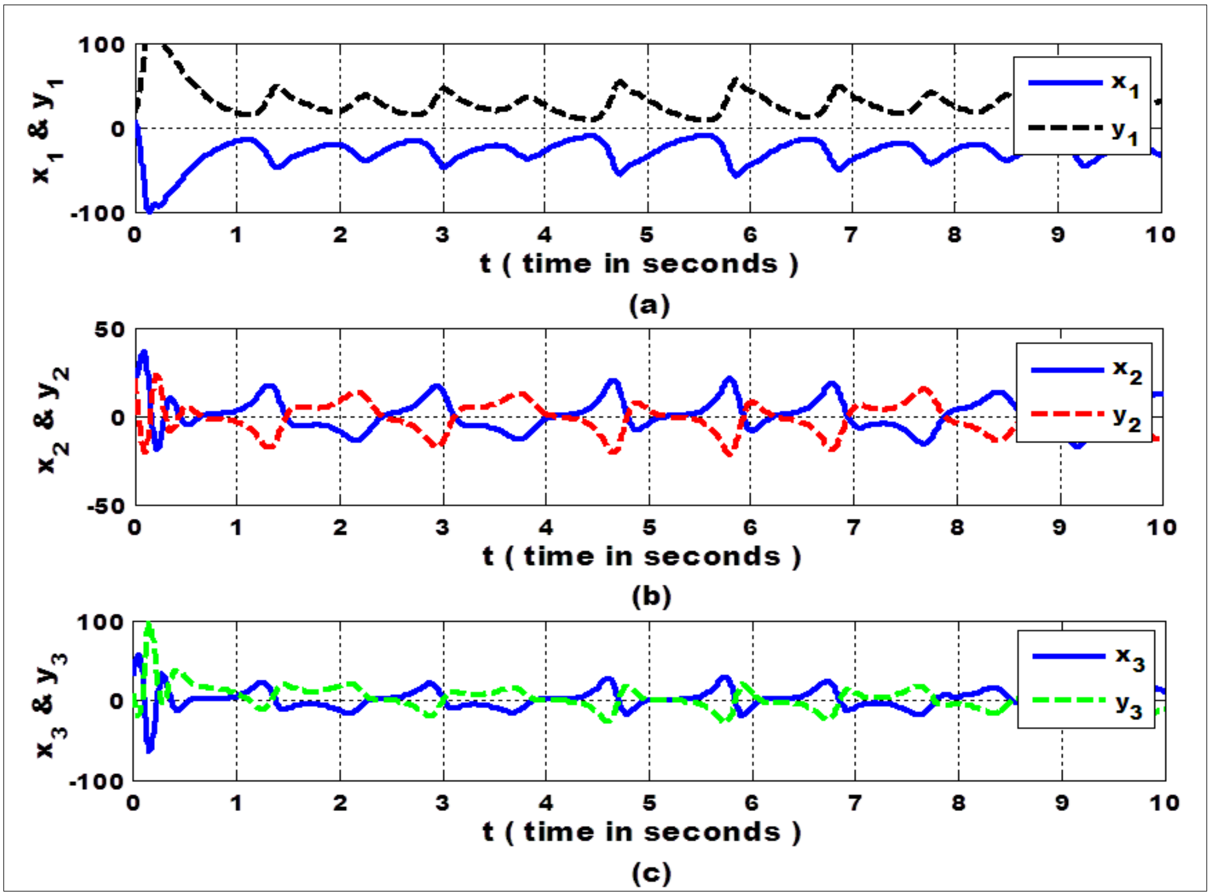

Fig. 3: *Anti-synchronization Between States of Master (1) and Slave (2) Systems.*

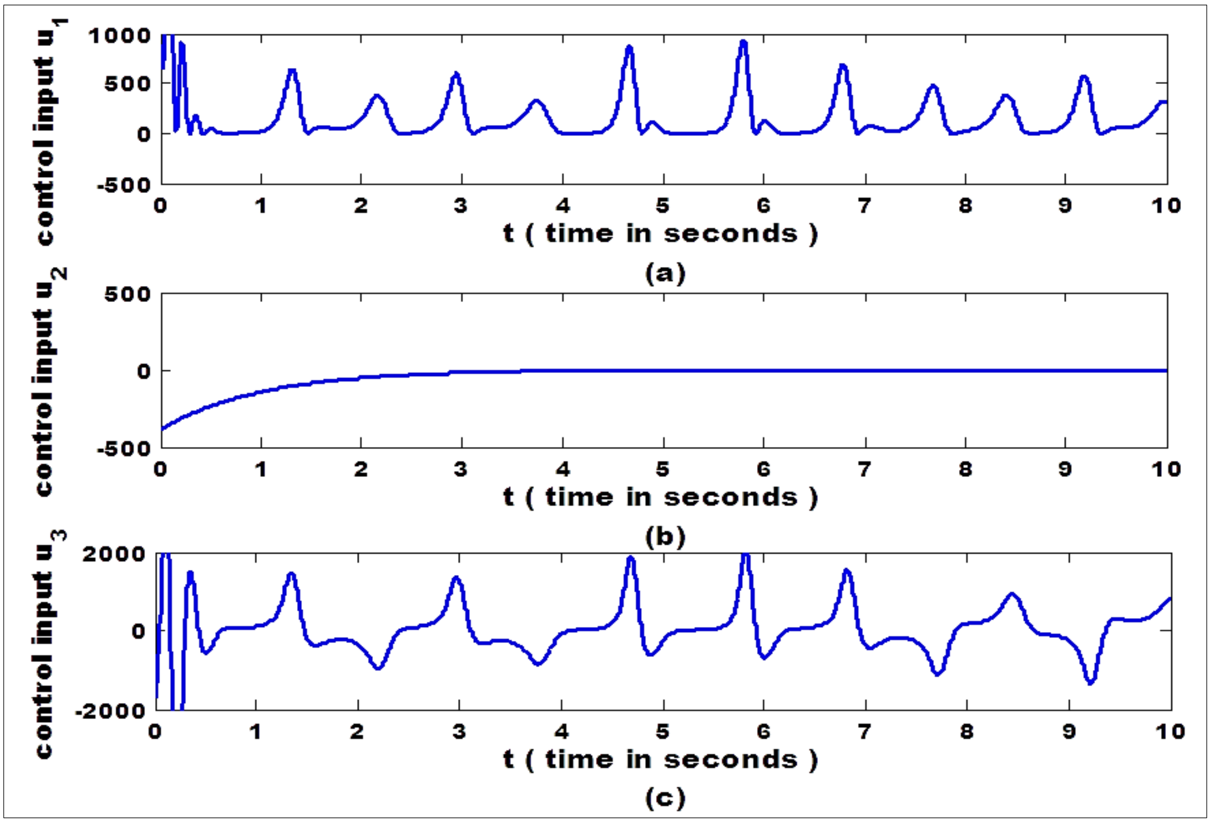

Fig. 4: *Time Response of Control Inputs to Anti-synchronize Master and Slave States.*

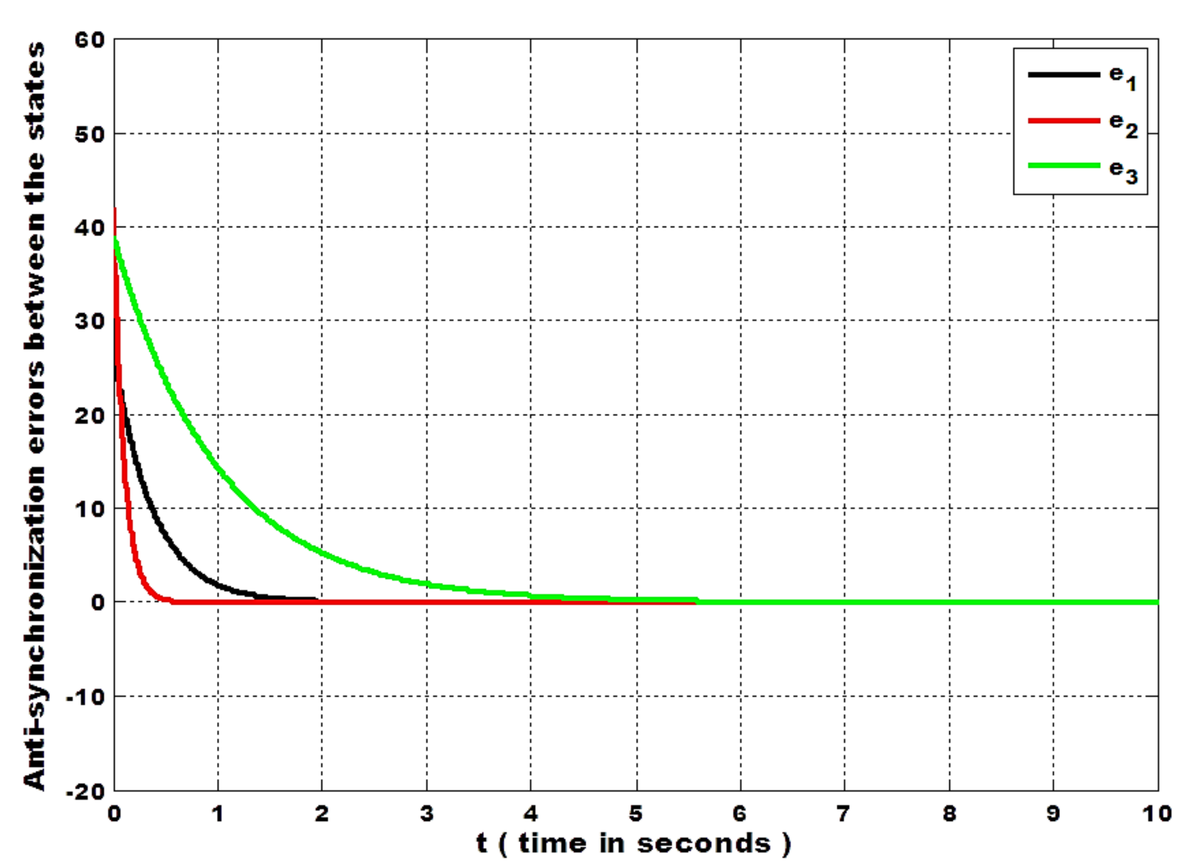

***Fig. 5:** Anti-synchronization Errors Between the States of Master (1) and Slave (2) Systems.*

RESULTS AND DISCUSSION

We are using fourth-order Runga-Kutta method for solving the dynamics Eqs. (1), (2) and nonlinear active controller and simulating the result with normalized time step h=0.005.

For chaotic behavior of Bhalekar–Gejji system, parameters are selected as $\omega = -2.667, \mu = 10, \alpha = 27.3, b = 1$ [23]. The initial condition for plotting anti-synchronized responses of states, controller, error dynamics between master system Eq. (1) and slave system Eq. (2), are given by,

$x_m(0) = [x_1(0) \quad x_2(0) \quad x_3(0)\,]^T = [10 \quad 20 \quad 30]^T$ and $y_s(0) = [17 \quad 22 \quad 9]^T$.

State response of master and slave systems is given in Figure 2 and anti-synchronization between states of master system Eq. (1) and slave system Eq. (2) are shown in Figure 3 which reveals successful achievement of the proposed objective using proposed controller as shown in Figure 4. Finally, anti-synchronization errors are given in Figure 5 between the states of master and slave systems.

CONCLUSION

In this paper anti-synchronization scheme is presented to investigate the anti-synchronization problem of developed Bhalekar–Gejji chaotic system. A nonlinear active controller has been proposed to guarantee the occurrence of global asymptotically stability. It has been shown that the master and slave systems are synchronized by proper design of control law. Finally, simulation results establish feasibility and effectiveness of proposed theoretical design. The proposed anti-synchronization can be used for the purpose of secure communication.

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