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Dependence of Fidelity of Quantum Teleportation for Pure State on Degree of Entanglement

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ABSTRACT

Quantum information and computation has been a rapidly evolving field. As Landauer pointed out, information is physical, so it should not look strange to try to bring together quantum mechanics and information theory. Indeed, it was soon realized that it was possible to use the laws of quantum physics to perform tasks which are unconceivable within the framework of classical physics like quantum teleportation, superdense coding, quantum cryptography, Shor's factorization algorithm or Grover's searching algorithm. Here, quantum teleportation has been discussed in a different way for two qubit pure state. It is a well-known fact that any two qubit pure states can be decomposed into two parts – pure entangled part and factorizable part. The square of the weightage of entangled part (p) gives the amount of entanglement in the state. It has been shown that the fidelity of quantum teleportation is a function of p and its variation with p has been plotted for all the possible outcomes.

Keywords: quantum mechanics, teleportation, Fidelity

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1. INTRODUCTION

Entanglement is an elementary static resource of quantum mechanics. Its properties are amazingly different from classical resources. There are too many applications of entanglement like quantum key distribution, quantum dense coding, entanglement swapping, and quantum repeaters. Besides these, quantum information theory – quantum teleportation, quantum cryptography [1], quantum tomography [2, 3] and quantum computation are most common. Since entanglement is the stepping stone of quantum information theory, it is also playing an important role in quantum computation. Quantum teleportation has been discussed by Prakash et al. [4] for maximally and non-maximally entangled states and they derived a relation between minimum assured fidelity and the concurrence of the state. Takeoka et al. [5] analyzed the continuous variable quantum teleportation and also discussed the same in terms of quasi-probability distribution.

It is recognized that the information content of an array of qubits depend not only on the states of the individual qubits but also on whether or not the qubits are entangled among themselves. It has been shown that a maximally entangled array of qubits has greater information content than an equivalent array in which the qubits are not or are partially entangled.

In Section 2, we briefly examine some of key motivations and ideas underlying quantum teleportation. In Section 3, quantum teleportation using partially entangled state is studied and fidelity of the process is calculated.

2. QUANTUM TELEPORTATION PROTOCOL

Let us imagine that sender Alice wants to transfer a quantum state to a distant receiver Bob and cannot use quantum communication channel. Then she has to use classical communication channel through which a bit can be sent at a time; it implies no transfer of quantum state. As well as, it is restricted by the postulate of quantum mechanics which reveals that any measurement performed by sender on the state in its possession will destroy the quantum state at hand without revealing all the necessary information for the receiver to reconstruct it. The problem is resolved by the use of quantum entanglement. Bennett et al. [6] gave the basic idea of quantum teleportation to transfer the state of a quantum system to another system at distant location. A qubit in an unknown state cannot be cloned [7, 8] which indicates no cloning of state during QT process. In fact the state at sender side disappears as it reaches at receiver's side. The protocol is as follows:

1. The sender and receiver share an entangled state of two particles.
2. One of the entangled particles is carried by Alice and other by Bob.
3. Alice has two states – one is entangled state and the other is being teleported.
4. The joint state of Alice can be expressed by the superposition of these states. This can be converted into Bell basis since Bell state forms a complete orthogonal basis.
5. Since Bob and Alice share an entangled state, any change at the Alice part will be propagated to Bob.
6. Now Alice performs a Bell state measurement (BSM) on joint state, that is, she projects her state onto one of the four Bell states.
7. Alice sends the result of BSM which is of a bit to Bob via classical communication channel.
8. After getting the result, Bob performs the appropriate unitary transformation on its particle to retrieve the state which initially Alice wants to teleport.

3. QUANTUM TELEPORTATION OF A STATE USING PARTIALLY ENTANGLED STATE

Any pure state can be decomposed into maximally entangled and factorizable part [9]. Let Alice and Bob share partially entangled state given by

$$|\psi\rangle_{23} = p|\psi_e\rangle_{23} + \sqrt{1-p^2}|\psi_f\rangle_{23}, \quad (3.1)$$

with

$$|\psi_e\rangle_{23} = \frac{1}{\sqrt{2}}\left(|00\rangle_{23}|11\rangle_{23}\right) \quad (3.2a)$$

and

$$|\psi_f\rangle_{23} = \alpha_1|00\rangle_{23} + \alpha_2|01\rangle_{23} + \alpha_3|10\rangle_{23} + \alpha_4|11\rangle_{23}, \quad (3.2b)$$

with $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 = 1$ and $\alpha_1\alpha_4 = \alpha_2\alpha_3$.

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The state to be teleported is,

$|\psi\rangle_1 = a|0\rangle_1 + b|1\rangle_1$. (3.3)

At Alice's end, the state of the composite system is,

$|\psi\rangle = |\psi\rangle_1 \otimes |\psi\rangle_{23}$

$= |\psi'\rangle + |\psi''\rangle$, (3.4)

Where,

$|\psi'\rangle = p\big(a|0\rangle_1 + b|1\rangle_1\big)|\psi_e\rangle_{23}$ (3.5a)

and

$$|\psi''\rangle = (1-p^2)^{1/2}\big(a|0\rangle_1 + b|1\rangle_1\big)|\psi_f\rangle_{23}. \quad (3.5b)$$

Now, Alice makes a BSM on the state in her possession,

$$|\psi\rangle = \sum_{r=1}^{4} |Br\rangle_{12}\langle Br|\psi\rangle$$

$$= \sum_{r=1}^{4} |Br\rangle_{12}\langle Br|\psi'\rangle + \sum_{r=1}^{4} |Br\rangle_{12}\langle Br|\psi''\rangle, \quad (3.6)$$

The result of BSM will be one of the four Bell states and therefore there exist four cases. Out of these four, we have observed that the fidelity has two values – one for two cases and other for reaming two cases. The fidelity of teleportation is, given by

$F = |\langle\psi_1|out\rangle|^2$

here $|\psi_1\rangle = |\psi\rangle_1$ and $|out\rangle$ be the state received by Bob. Here, two different cases are discussed in detail.

Case I: If the BSM result is $|B1\rangle_{12} = \frac{1}{\sqrt{2}}\big(|00\rangle_{12} + |11\rangle_{12}\big)$ then after simplification, we get

$$F = \frac{1}{x^2+y^2}[ax+by]^2. \quad (3.7)$$

Here,

$$x = \left(\frac{p}{2}a + \left(\frac{1-p^2}{2}\right)^{1/2}(a\alpha_1 + b\alpha_3)\right) \quad (3.8a) \text{ and}$$

$$y = \left(\frac{p}{2}b + \left(\frac{1-p^2}{2}\right)^{1/2}(a\alpha_2 + b\alpha_4)\right). \quad (3.8b)$$

It is evident from Eq. (3.7) that fidelity depends on the values of $\alpha_1, \alpha_2, \alpha_3, \alpha_4, a$ & b. Normalization of the state imposes conditions on parameters which are $\alpha_1\alpha_4 = \alpha_2\alpha_3$ & $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 = 1$. A graph (Fig. 1) has been plotted between fidelity and p when $\alpha_1^2 = 0.3 = \alpha_3^2$; $\alpha_2^2 = 0.2 = \alpha_4^2$.

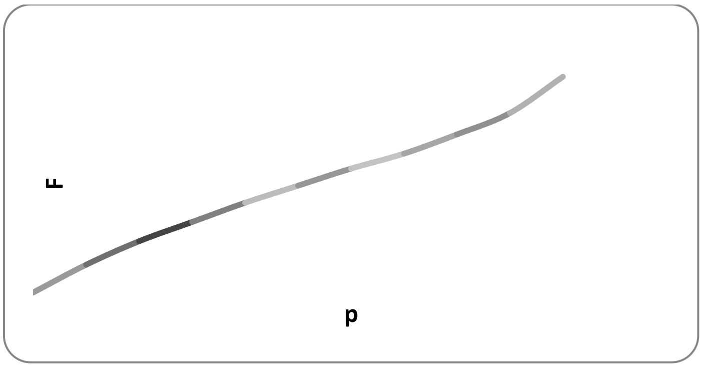

Fig.1: *Variation of Fidelity with p.*

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Case II: If the BSM result is $|B2\rangle_{12} = \frac{1}{\sqrt{2}}(|00\rangle_{12} - |11\rangle_{12})$ then the fidelity of teleportation comes out,

$$F = \frac{1}{x^2 + y^2}[ax + by]^2 \qquad (3.9)$$

with

$$x = \left(\frac{p}{2}a + \left(\frac{1-p^2}{2}\right)^{1/2}(a\alpha_1 - b\alpha_3)\right) \qquad (3.10a)$$

and

$$y = -\left(\frac{-p}{2}b + \left(\frac{1-p^2}{2}\right)^{1/2}(a\alpha_2 - b\alpha_4)\right). \qquad (3.10b)$$

A graph (Fig. 2) has been plotted between fidelity and p when $\alpha_1^2 = 0.3 = \alpha_2^2$; $\alpha_3^2 = 0.2 = \alpha_4^2$.

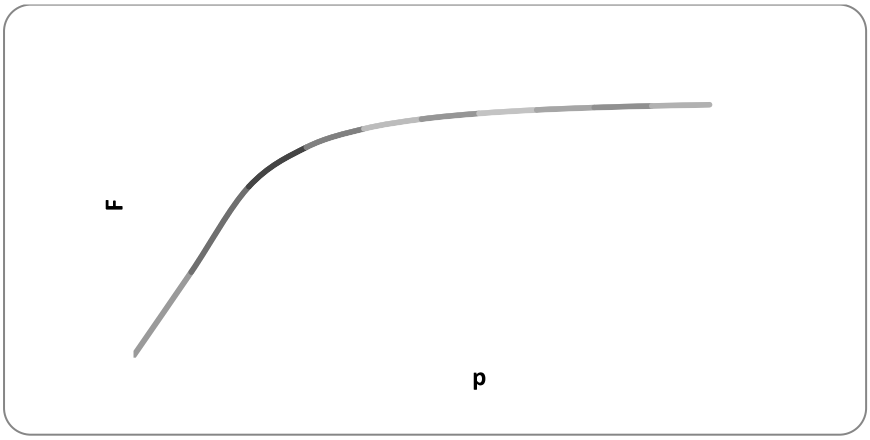

***Fig.2:** Variation of Fidelity with p.*

If the BSM result is $|B3\rangle_{12} = \frac{1}{\sqrt{2}}(|01\rangle_{12} + |10\rangle_{12})$, we get the same expression for fidelity as obtained in **Case I** [Eq. (3.7)] and for the BSM result $|B4\rangle_{12} = \frac{1}{\sqrt{2}}(|01\rangle_{12} - |10\rangle_{12})$, the fidelity is same as found in Case II [Eq. (3.9)].

4. CONCLUSIONS

It is evident from Case I and Case II that fidelity depends on the amount of entanglement present in the state. Also, it has been obtained that the fidelity has greater value in Case I in general.

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