Research & Reviews: Journal of Physics
Volume 2, Issue 1, ISSN: 2278-2265

STM JOURNALS

Breakdown of Physicothermodynamic System of The Human Neural Network in The Fractional Dimensional Space Due To Z+ Bis Intake: Introduction to Photonic & Phononic Nociceptions

Tuhin Dutta[1]*, Deepak Kumar[2], Madan Mohan Bajaj[3], Sobinder Singh[4]
[1]Department of Physics, Kirori Mal College, University of Delhi, Delhi, India
[2]Department of Physics, Singhania University, Rajasthan, India
[3]Department of Physics & Astrophysics, University of Delhi, Delhi, India
[4]Department of Applied Sciences, Maharaja Surajmal Institute of Technology, Guru Gobind Singh Indraprastha University, Delhi, India

Abstract

Nociceptionics deals with the properties, dynamism of nonciceptons, emitted from the quasi-particles of pain perception due to killing or annihilation of living organism in any part of the globe. The new subject of Nonciceptonics is analogous to the conventional discipline of Digital & Pulse Electronics and Abstract Algebra. In our present work we will study the characteristics of fractional dimensional space in the human brain as a severe consequence of the nociceptons emitted from the living organism while being killed or annihilated. We shall provide the relevant numerical calculations using the various characteristics of nociceptons. We have explained our views in half dimensional space.

Keywords: *nociceptionics, nonciceptons, chemical Potential, fermi gas*

PACS: *05.70.-a, 82.39.-k, 87.15.R-, 71.10.Ca*

****Author for Correspondence*** E-mail: tuhindutta38@gmail.com

INTRODUCTION

Half dimensional space originates in the following way:

This space is not complete. It is fractional. Fractional dimensional space bears by meeting material and mind sciences. We cannot show mind science on the table. We cannot keep our affection or anger in the form. It is half dimensional we know about:

(i) One dimensional space (Straight line)
(ii) Two dimensional space (Paper)
(iii) Three dimensional space (Box, Table etc.)
(iv) Four dimensional space (X, Y, Z, etc.)

So there should also be a fractional space = $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ etc. This space is not just like integral dimensional space.

CHEMICAL POTENTIAL & SECOND LAW OF THERMODYNAMICS OF THE NOCICEPTONS

The universe in which we all live is a many particle system. Chemical potential characterizes the many particle system in thermal equilibrium [1–6].

Suppose the change in the total energy of the system – dU
No. of Nociceptons exchanged = dN
Temperature of the reservoir in the absolute scale = T
Pressure of the reservoir = T
Change in entropy by the temperature T = dS
Change in the Volume (by the pressure p) = dV
Chemical Potential = μ
Pressure = P
Entropy = S
Change in entropy = dS

Then according to the second law of thermodynamics

$dU = Tds - pdv + UdN$

When S & V of the system are fixed, ds = 0, dv=0

$du = udN$

$$\text{or } \mu = \frac{dU}{dN}$$

$$\text{or } \mu = \frac{dU}{dN}\Big|_{S,V} = \left[\frac{dU}{dN}\right]_{S,V} \quad \text{are fixed} \qquad (1)$$

NOCICEPTION	=	Perception of Pain
NOCICEPTION WAVES	=	Pain Waves or E.P. Waves (Einsteinian Pain wave)
	=	VLF signal (Very Low frequency signal)
	=	SHOCK WAVE
	=	Electromagnetic Signal

In the ground state, S vanishes

CHEMICAL POTENTIAL IN FRACTIONAL AND MULTIDIMENSIONAL SPACE

Various techniques are described in the literature for evaluating μ of the non-interacting Fermi gas at low-temperature limit. In 1928, Sommerfeld obtained μ as a low-temperature expansion in the three-dimensional systems. This method is not correct for an arbitrary value of μ/k_BT (k_B is the Boltzmann constant) because this method omits the term exp $(-\mu/k_BT)$ in the calculation. However, this method is accurate in the low-temperature limit as exp $(-\mu/k_BT)$ vanishes in this limit. The Sommerfeld method can be easily extended to calculate several ground state properties of the free fermion gas using the thermodynamic relations. Both μ and U in 3D are calculated upto order T^6 by Kiess and up to order T^8 by Aguilera-Navarro et al. The only limitation of the Sommerfeld method is that it cannot be used for two-dimensional (2D) systems. However, the evaluations of μ and U in the 2D system is analytic and the expressions for these quantities are obtained in closed forms. The other two procedures are the Cauchy's integral method and polylogarithm method. The results obtained in these methods are identical to those obtained in the Sommerfeld method [7–16].

The standard Pade approximant technique reproduces the correct behaviour of a function for which only a few terms are available. In this technique, the truncated series is expressed as a ratio of the two polynomials of finite sizes so that the ratio is convergent. Using this method, the truncated series of μ and U valid in the low-temperature and high density regimes are extended to intermediate-temperature and low-density regimes [15–18].

The multidimensional space method serves as a model for studying the dimensional dependence of the physical properties. For example, the properties of polaron are obtained as a function of n, an integer specifying the dimensions. The evaluation of μ and U is this space has been attempted by Cetina et al. Using the series expension method. It is more elegantly evaluated using the Cauchy's contour integration method and the polylogarithm method [19–22]. All these methods are found to give identical results. In addition to these, the polylogarithm method in this space was employed to calculate μ in the one-dimensional (1D) and zero-dimensional (0D) systems.

The quantum well and quantum wire fabricated using the semiconductor hetero-junctions are 3D systems with embedded 2D and 1D structures, respectively.

In 1986, Ishida theoretically noticed the dimensional cross-over of the plasmon from 2D to 3D when the well width of the quantum well is increased. A narrow quantum well has only one level occupied by the electron so that the system manifests 2D behaviour. However, the level separation decreases with the increase of the well width so that higher and higher levels are occupied by electrons. When the well width is very large, the system shows 3D behaviour. Consequently, the system shows fractional-dimensional (FD) behaviour with the dimension varying between 2 and 3 when the well width is finite [22–27]. Similarly, the

quantum wire with finite internal area shows FD behaviour with its dimensionality varying between 1 and 3.

FRACTIONAL DIMENSIONAL SPACE OR DYNAMIC SPACE IN THE HUMAN BRAIN

The Fractional Dimensional space is termed as dynamic space. It differs from the geometric space in that its dimensionality is determined by physical interactions are seen from the viewpoint of the excitation dynamics.

The anisotropic low-dimensional space in 3D structures becomes isotropic in the FD space and the dimension provides a measure for the degree of anisotropy in the actual physical system.

The dimension is determined from the extension of radius of bosons such as exciton, polaron, exciton-polaron and plasmon with respect to the well width in quantum well and the square root of the internal area in quantum wire. The quasi-2D layered structures of high-temperature superconducting thin films are actually not 2D objects, but fractals with Hausdorff dimensionalities between 2D and 3D in the real laboratory situation.

The formalism of He has been used to describe the superconductivity and the transport properties in fractals. An accurate evaluation of chemical potential μ is necessary to find the temperature dependence of the excitation spectra in these systems.

In the present work, we have used the Sommerfeld method to find μ and U in Fractional Dimensional space at low-temperature and high-density limits. The results are fitted to Pade approximant forms to extend their validity to intermediate-temperature and low-density limits.

METHOD OF CALCULATION

An interpolating formula for μ of the noninteracting Fermi gas in the Fractional Dimensional (FD) space valid at high temperatures at an arbitrary density of nociceptons. In the present work, we will present the derivation of μ valid for Low temperature and high-density limits only.

The number of nociceptons

$$N = 2\sum_K \frac{1}{\exp((\varepsilon_k - \mu)/(K_B T) + 1} \qquad (2)$$

$$N = 2\sum_K \frac{1}{\left[e\frac{e_k - \mu}{k_B T} + 1 \right]}$$

Here $\varepsilon_K = \frac{h^2 k^2}{2m}$ is the energy variable with m; m is the effective mass of nocicepton. The factor 2 accounts for the spin degeneracy.

The internal energy

$$U = 2\sum_K \frac{1}{\exp((\varepsilon_k - \mu)/(K_B T) + 1} \qquad (3)$$

Here, K_B is the Boltzmann constant, T is the temperature in the absolute scale, U is the total energy, and ε_K is the kinetic energy.

In the FD Space, K is not a vector space. The coordinates in the FD space are termed as pseudo coordinates. The vector operations are not allowed in this space. Since the FD space is taken as isotropic in the method of calculation. The sum over K can be transformed into integral over positive K as:

$$\sum_K ..... = \frac{V}{(2\pi)^\alpha} \frac{2\pi^{(\alpha-1)/2}}{\Gamma((\alpha-1)/2)} \int k^{\alpha-1} dk \int_0^\pi d\theta \sin_\theta^{\alpha-2} \qquad (4)$$

where V is the volume in FD space and Γ (x) is the Euler's gamma function. This transformation is also the same as in the multidimensional space where α is replaced by n. This transformation is not valid for the 1 D system since Γ (0) = ∞. where $\infty \to 1$. This formalism can be applied to quantum dots. Since Γ [(α–1)/2] = $-2\sqrt{\pi}$ when α=0 At T=0 using Eq. (4) in Eq. (2) and carrying out the integration over θ, we find

Density of nociceptons =

$$\rho = 2\rho_0 \int_0^{E_F} \varepsilon_K^{(\alpha-2)/2} d\varepsilon_K \qquad (5)$$

The density of nociceptons $\rho = \frac{N}{V}$

$$\rho_0 = (m/2\pi h^2)^{\alpha/2} / \Gamma(\alpha/2)$$

Here $\Gamma[(\alpha-1)/2]$ in Eq. (5) gets cancelled after Q integration except when α=1. Carrying out integration in Eq. (5)

Fermi Energy $E_F = \left(\frac{\alpha_p}{4\rho_o}\right)^{2/\alpha} = \frac{h^2}{2m}\left(\frac{(4\pi)^{\alpha/2}\Gamma(1+\frac{\alpha}{2})\rho}{2}\right)^{2/\alpha}$ (6)

$$K_F = ((4\pi)^{\alpha/2}\Gamma(1+\alpha/2)\rho/2)^{1/\alpha}.For T \neq 0$$

So density of Nociceptons ejected due to BIS processes

$$\rho = 2\rho_0 \int_0^\infty \frac{\varepsilon_K{}^{(\alpha-2)/2}}{\exp((\varepsilon_k - \mu)/K_B T)+1} d\varepsilon_K \quad (7)$$

For α=2, the integration in Eq. (7) is carried out exactly, then

$$\rho = 2\rho_0 \left[\mu + K_B T \log\left\{1+\exp\left(\frac{\mu}{K_B T}\right)\right\}\right] \quad (8)$$

The Fermi energy

$$E_F = \mu + K_B T \log\left[1+\exp\left(\frac{\mu}{K_B T}\right)\right] \quad (9)$$

Divide both sides by E_F,

$$1 = \frac{\mu}{E_F} + \frac{K_B T}{E_F}\log\left[1+\exp\left(\frac{\mu}{K_B T}\right)\right]$$

$$\frac{\mu}{E_F} = 1 - \frac{K_B T}{E_F}\log\left[1+e^{(\mu/K_B T)}\right]$$

$$or\ \frac{\mu}{E_F} = 1 + \frac{K_B T}{E_F}\left\{\log\left(1+e^{(\mu/K_B T)}\right)^{-1}\right\} \quad (10)$$

At absolute zero (i.e., at T=0)

$$E_F = 1 + \frac{K_B T}{E_F}\log\left[1-e^{\mu/K_B T}\right]$$

Taking the low temperature expansion in powers of $K_B T$,

$$E_F = 1 + \frac{K_B T}{E_F}\log\left[1-e^{\mu/K_B T}\right]$$

$$\frac{\mu}{E_F} = 1 + \frac{K_B T}{E_F}\log\left[1-e^{E_F/KT}\right]$$

When T=0, $\mu=E_F$

The sommerfeld expansion for the density (p) of nociceptons is

$$\rho = \frac{4\rho_0}{\alpha}\left[\mu^{\alpha/2} + \sum_{l=1}^{\infty}(K_B T)^{21}\varsigma(2l)\frac{4^l - 2}{4l}\frac{d^{2l-1}\mu^{\frac{\alpha}{2}-1}}{d\mu^{2l-1}}\right] \quad (11)$$

where ς (x) – the Riemann zeta function.

The Fermi energy

$$E_F = \mu\left[1+\frac{\alpha-2}{12}\left(\frac{\pi k_B T}{\mu}\right)^2 + \frac{(\alpha-2)(\alpha^2-25\alpha+74)}{1440}\left(\frac{\pi k_B T}{\mu}\right)^4 + ...\right]$$

$$+\frac{(\alpha-2)(4\alpha^4-259\alpha^3+4419\alpha^2-23996\alpha+41252)}{362880}\left(\frac{\pi k_B T}{\mu}\right)^6$$

$$+\frac{\left\{\begin{matrix}(\alpha-2)(18\alpha^6-2167\alpha^5+81924\alpha^4-1303409\alpha^3\\+9638270\alpha^2-33428844\alpha+44072968)\end{matrix}\right\}}{87091200} x\left(\frac{\pi k_B T}{\mu}\right)^8 + ...] \quad (12)$$

For finding μ when α≠2, the inversion of Eq. (12) is carried out using the isotherms approach. In the approach the number of particles does not depend upon temperature at a given volume. Therefore, E_F is independent of temperature,

$$\left(\frac{\partial E_F}{\partial T}\right)_V = 0 \quad (13)$$

A series expansion of μ/E_F in terms of K_BT/E_F where μ is the chemical Potential and E_F is the Fermi Energy

$$= \frac{\mu}{E_F} = 1 + a_2\left(\frac{k_BT}{E_F}\right)^2 + a_4\left(\frac{k_BT}{E_F}\right)^4 + a_6\left(\frac{k_BT}{E_F}\right)^6 + a_8\left(\frac{k_BT}{E_F}\right)^8 + \ldots\ldots \quad (14)$$

Here, a_2, a_4,… etc. are the coefficients of expansion.

The coefficients of odd powers of T=0, because E_F contains only even powers of T. These coefficients are

$$a_2 = -\frac{(\alpha-2)\pi^2}{12}$$

$$a_4 = -\frac{(\alpha-2)(\alpha-6)(\alpha-9)\pi^4}{1440}$$

$$a_6 = -\frac{(\alpha-2)(\alpha-4.2231)(\alpha-8.6476)(\alpha-10)(\alpha-20.8794)\pi^6}{90720}$$

$$a_8 = -\frac{\left\{(\alpha-2)(\alpha-4.0247)(\alpha-6.3142)(\alpha-9.5286)\right\}}{\left\{.(\alpha-14)(\alpha-14.8860)(\alpha-37.9687)\pi^8\right\}} \quad (15)$$

$$4838400$$

For quantum wire, $\mu/E_F = 1+(\pi^2/12)(K_BT/E_F)^2$ At $\alpha \to 1$ For α>2, we find that μ/E_F decreases T. When α<2, μ/E_F increased with T which is unphysical as $\mu/E_F \to \infty$ when $T \to \infty$.

Grether et al. [27] have shown that μ/E_F increases in the low-temperature limit and then becomes negative at $T \to \infty$ with a hump in the intermediate temperature limit.

The internal energy U can be calculated using the general thermodynamic relationship [4]

$$(\alpha+2)U - 2T\left(\frac{\partial U}{\partial T}\right)_{v,N} - \alpha N\mu + \alpha NT\left(\frac{\partial U}{\partial T}\right)_{v,N} = 0 \quad (16)$$

A series expansion of U

$$U = U_0\left[1 + A_2\left(\frac{k_BT}{E_F}\right)^2 + A_4\left(\frac{k_BT}{E_F}\right)^4 + A_6\left(\frac{k_BT}{E_F}\right)^6 + A_8\left(\frac{k_BT}{E_F}\right)^8 + \ldots\ldots\right], \quad (17)$$

Here U_0 = the internal energy at T=0, A_2, A_4 etc. are the coefficients to be determined. These coefficients are obtained by substituting Eqs. (17) and (14) in Eq. (16),

$U_0 = \alpha NE_F/(\alpha+2)$,

$$A_2 = \frac{(\alpha+2)\pi^2}{12},$$

$$A_4 = \frac{(\alpha+2)(\alpha-2)(\alpha-9)\pi^4}{480},$$

$$A_6 = \frac{(\alpha+2)(\alpha-2)(\alpha-4.2231)(\alpha-8.6476)(\alpha-20.8794)\pi^6}{18144},$$

$$A_8 = \frac{\left\{7(\alpha+2)(\alpha-2)(\alpha-4.0247)(\alpha-6.3142)x \right\}}{\left\{(\alpha-9.5286)(\alpha-14.8860)(\alpha-37.9687)\pi^8\right\}}, \quad 691200 \tag{18}$$

Substituting $E_F = (\alpha N/4n_0V)^{2/\alpha}$ in Eq. (17) and using Eq. (1), we can obtain Eq. (14). The internal energy increases for all α.

PADE APPROXIMANT METHOD

The truncated series for μ and U correctly finds the low-temperature limit of the numerical method. We need to express the series as a ratio of two finite sized polynomials in the Pade approximant technique for extending their validity to intermediate temperature and relatively low-density regimes. This method has been described by Baker [9]. As pointed out earlier, this method has been applied to truncated series of μ/E_F and U/U_0 in 3D containing terms up to the 8^{th} power of T [5], where different Pade approximant forms have been fitted to the series expansions. Among three different forms, [0/3], [2/1] and [1/2], [0/3] form has been found to be the best. In order to include a_8 term in the fitting, the entire series in the present work is expressed in terms [0/4], [1/3], [2/2] and [3/1] forms [1/3] is the best among them. Using this form, μ/E_F is expressed as:

$$\frac{\mu}{E_F} = \frac{1+p_2(k_BT/E_F)^2}{1+q_2(k_BT/E_F)^2+q_4(k_BT/E_F)^4+q_6(k_BT/E_F)^6} \tag{19}$$

where the coefficients are obtained using the Mathematical package [28] as

$p_2 = (a_2^4 - 3a_2^2a_4 + a_4^2 + 2a_2a_6 - a_8)/A, q_2 = (a_2^4 - a_2^2a_4 + a_2a_6 - a_8)/A, q_4 = (a_2^4 - a_2^2a_4 + a_2a_6 - a_8)/A$ and $q_6 = (-a_3^4 + 2a_2a_4a_6 - a_6^2 - a_2^2a_8 + a_4a_8)/A, where A = a_2^3 - 2a_2a_4 + a_6$. These coefficients are evaluated using Eq. (15).

Similarly, U is expressed in the [1/3] Pade approximant form as

$$\frac{U}{U_0} = \frac{1+P_2(k_BT/E_F)^2}{1+Q_2(k_BT/E_F)^4+Q_4(k_BT/E_F)^6+Q_6(k_BT/E_F)^8} \tag{20}$$

where P_2, Q_2, Q_4 and Q_6 are related to A_2, A_4, A_6 and A_8 and in μ/k_BT.

RESULTS AND DISCUSSIONS

We have calculated μ/E_F as a function to k_BT/E_F numerically at a constant density for α=1.01, 1.5, 2.5 and 3 and compared them with those obtained in the Sommerfeld and Pade approximant methods in Figure 1. The numerical results also show similar behaviour as in the other two methods at extremely low-temperature limit. For $\alpha \geq 2$, we find the μ/E_F decreases with *T*. On the other hand, for $\alpha<2$, μ/E_F increases with *T* which is unphysical as $\mu/E_F \to -\infty$ when $T \to -\infty$ Grether et al [27], in the numerical method, have shown that μ/E_F increases in the low-temperature limit and then becomes negative at $T \to -\infty$ with a hump in the intermediate temperature limit. The overall behaviour of the temperature dependence of μ in 3D has been described by Cook and Dickerson [29]. At zero temperature, the system is in the ground state with zero entropy. At a small *T* the sharpness of the edge of the Fermi surface is lost. In the Sommerfeld expansion method the entropy up to order T^3 in FD space is obtained as

$$\frac{S}{\alpha N k_B} = \frac{\pi^2}{6}\left(\frac{k_BT}{E_F}\right)\left[1+\frac{(\alpha-2)(\alpha-9)\pi^2}{20}\left(\frac{k_BT}{E_F}\right)^2+.....\right] \tag{21}$$

The second term is negative for $\alpha>2$ and it is positive for $\alpha<2$. As the temperature rises, the total internal energy of the system increases and some of the fermions begin to occupy excited states. In order to keep entropy as zero, the added fermions for $\alpha \geq 2$ must go into the low-lying vacant single-particle states little below E_F left open by the excited fermions.

The number of available microstates must be minimized by cooling the gas. The change in internal energy of the Fermi gas must be positive, but a little below E_F. As the temperature increases, more of the low-lying states become vacant. To add a new particle without increasing the entropy, it requires the new particle to go into a low-lying single-particle state, considerably well below E_F while once again cooling the gas slightly to avoid an increase in the number of microstates. However, a particle in system with $\alpha<2$ is prevented from going below E_F as its paths are blocked by other excited particles due to greater entropy then in $\alpha\geq2$. This suggests that the particles which are taken out of the Fermi sea are more dispersed for $\alpha<2$ resulting in greater entropy than in $\alpha\geq2$.

For $\alpha<2$, μ/E_F slowly rises with temperature compared to that in the Sommerfeld method. The comparison shows that the results in the Sommerfeld method fail to match with those obtained numerically beyond a certain temperature T_C and T_C decreases with decreasing α. The series expansion method is described reasonably well when $\alpha>2$. Figure 1 also shows that a fixed temperature μ is correctly calculated only when the density is high. For lower density there is no agreement between numerical and series expansion methods. As shown in Figure 1, the chemical potentials in the Pade approximant method are extended to higher temperatures and lower density of particles for $\alpha>2$. However, it does not give any substantial improvement for $\alpha<2$.

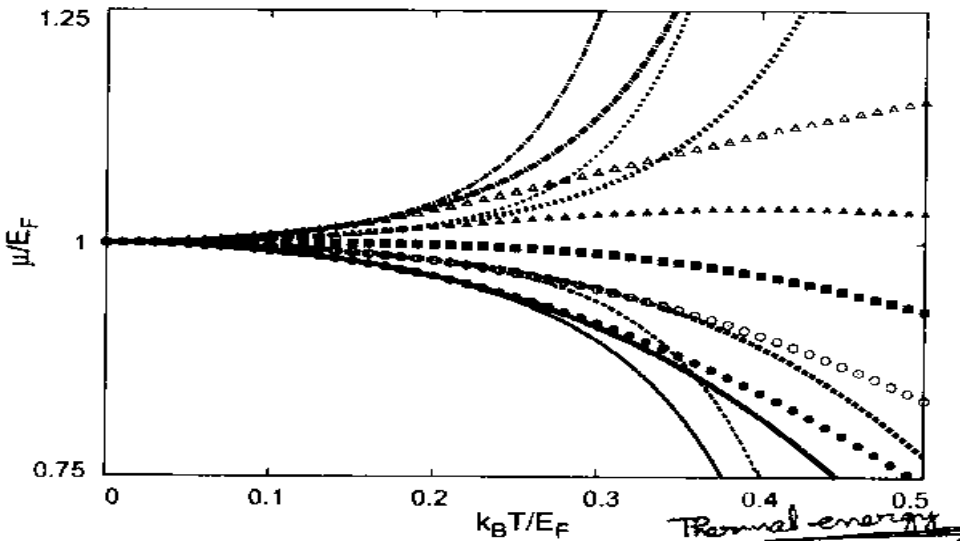

Fig. 1: *Chemical Potentials Scaled With Respect to Fermi Energy as a Function of Thermal Energy Scaled with the Same Fermi Energy for α=1.01, 1.5, 2, 2.5 and 3. The Chemical Potentials Calculated in The Numerical Method are Compared with Those Estimated in the Series Expansion Method and Pade Approximant Methods. In the Numerical Method: Closed Circles (α=3), Open Circles (α=2.5), Closed Squares (α=2), Closed Triangles (α=1.5) and Open Triangles (α=1.01). In the Series Expansion and Pade Approximant Methods: Solid Line (α=3), Dashed Line (α=2.5), Dotted Line (α=1.5) and Dot-Dashed Line (α=1.01). Here Thick and Thin Lines Correspond to Pade Approximant and Series Expansion Methods, Respectively.*

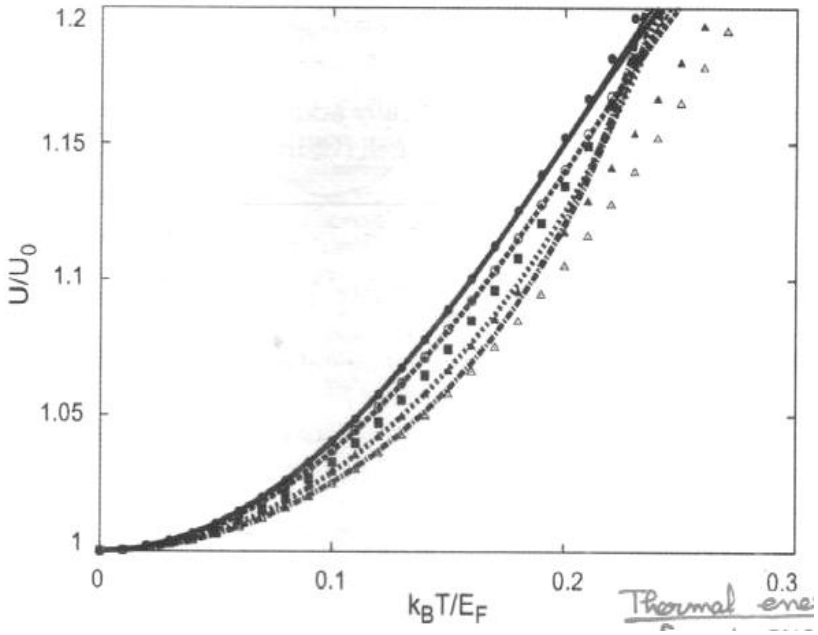

Fig. 2: *Internal Energy of Finite Temperature Scaled with Respect to Internal Energy at Zero Temperature as a Function of Thermal Energy Scaled with Fermi Energy for α=1.01, 1.5, 2, 2.5 and 3. The Internal Energies Calculated in the Numerical Method are Compared with Those Estimated in the Series Expansion and Pade Approximant Methods. The Symbols Carry the Same Meaning as in Figure 1.*

The internal energies obtained in Eq. (17) are compared with those obtained in the numerical method in Figure 2. Unlike μ, U is found to increase for all α values.

The agreement between the series expansion and numerical results is qualitatively the same as shown earlier for μ/E_F. The series expansion method is good for lower temperature and higher density of particles and it is better for $\alpha \geq 2$ than for $\alpha < 2$. The internal energies obtained in this method are compared with those obtained in the series expansion and numerical methods. While the Pade approximant method gives an improvement from low-temperature limit to intermediate-temperature limit and higher density of particles to lower density of particles when $\alpha > 2$, it is not so appreciable for dimensionalities below 2.

CONCLUSION

In summary, both μ and U of the noninteracting Fermi gas are calculated in the numerical and series expansion methods for α=1.01, 1.5, 2.5 and 3. Comparison of these quantities in the numerical and series expansion methods shows that the results obtained in the latter method are valid in the low-temperature and high-density regimes. These quantities in 2D are obtained in closed forms. In order to extend their validity to higher temperature and lower density, the results are expressed in the Pade approximant method. The [1/3] Pade approximant form is found to be the most suitable for the truncated series expansions of μ and U.

ACKNOWLEDGEMENT

The authors are grateful to the Vice Chancellor of University of Delhi and IP University for providing adequate laboratory and library facility in the campus.

REFERENCES

1. A L Fetter, J D Walecka, Quantum Theory of Many-Particle System, McGraw-Hill, New York, 1971.
2. R K Patheria, *Statistical Mechanics,* Pergamon, Oxford, 1972.
3. A Sommerfeld, *Z. Phys.1928;* 47, 1p.
4. E Kiess, *Am. J. Phys.* 1987; 55, 1006p.

5. V C Aguilera-Navarro, G A Estevez, W Salono-Torres, *Am. J. Phys.*1991; 59, 452p.
6. E Cetina, F Magana, A. A. Valladares, *Am. J. Phys.* 1977; 45, 960p.
7. (a) W A Barker, *J. Math. Phys.* 1986; (b) 1987; 27: 302p; (c) 1987; 28, 1385p.
8. M H Lee, *J. Math. Phys.*1995; 36, 1217p.
9. G A Baker, *Pade approximation (Encyclopedia and Mathematics and its Applications Part-1),* Addition-Wesley, Reading, MA, 1981.
10. A Chatterjee, *Phys. Lett.* 1989; A135, 380p.
11. M H Lee, *J. Math. Phys.* 1989; 30, 1837p.
12. A Apostol, *Phys. Rev.* 1997; E56, 4854p.
13. M H Lee, *Phys. Rev.*1996; E54, 946p.
14. H Ishida, *J. Phys. Soc.* Jpn. 1986; 55, 4396p.
15. X F He, *Phys. Rev.* 1991; B42, 2083p.
16. P Lefebvre, P Christol, H Mathieu, *Phys. Rev.* 1992; B46, 13603.
17. A Matos-Abiague, *Phys. Rev.* 2002; B65, 165321.
18. A Thilagam, A Matos-A biague, *J. Phys: Condens. Matter, 2004;* 16, 3981.
19. S Panda, B K Panda, *J. Phys.l: Condens. Matter* 2008; 20, 485201.
20. K F Karlson, M-A Dupertuis, H Weman, E Kapon, *Phys. Rev.* 2004; B70, 153306p.
21. L Salasnich, J. *Math. Phys.* 2000; 41, 8016p.
22. A S Alexandrov, *Physica,* 2001; C363, 231p.
23. Z Bak, *Phys. Rev.* 2003; b68, 064511.
24. T Zavada, N Siidland, R Kimmich, T F Nonnemmacher, *Phys. Rev.*1999; E60, 1292p.
25. F H Stillinger, *J. Math. Phys.* 1997; 18, 1224p.
26. T Vazifehshenas, S Ghasem, *Eur. Phys. J.* 2008; B65 1434p.
27. M Grether, M de Liano, M A Solis, *Eur. Phys. J.* 2003; D25, 287p.
28. S Wolfram, *The Mathematica Book*, Cambridge University Press: London; 1996.
29. G Cook, R H Dickerson, *Am. J. Phys.*1995; 63, 737p.